Think – Pair – Share

- Continuous vs. inhomogeneous media:
  - Q1: First of all, as a refresher, what are the three main types of microwave interaction with media that we discussed last time?
  - Q2: What fourth interaction type is irrelevant for continuous media but becomes relevant in inhomogeneous media?
  - Q3: Would you describe the atmosphere as a continuous or an inhomogeneous medium for microwave remote sensing and why?
Importance of Understanding EM Wave Interactions with the Atmosphere

- In some applications, we want to use microwave data to measure properties of the atmosphere
  - E.g. rain rates, cloud cover, water vapor content, ozone, CO2
- In some applications, we want to measure properties of the earth surface and need to know how the atmosphere changes the signal
- For simplicity’s sake, we will assume that the atmosphere is a largely homogeneous continuous medium

Transmissivity of the Atmosphere

<table>
<thead>
<tr>
<th>Frequency (GHz)</th>
<th>Wavelength (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>1000</td>
</tr>
<tr>
<td>0.1</td>
<td>100</td>
</tr>
<tr>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>10</td>
<td>1.0</td>
</tr>
<tr>
<td>100</td>
<td>0.1</td>
</tr>
<tr>
<td>1 GHz</td>
<td>30 cm</td>
</tr>
<tr>
<td>10 GHz</td>
<td>3 cm</td>
</tr>
<tr>
<td>100 GHz</td>
<td>3 mm</td>
</tr>
<tr>
<td>1 THz</td>
<td>0.3 mm</td>
</tr>
<tr>
<td>10 THz</td>
<td>30 μm</td>
</tr>
<tr>
<td>100 THz</td>
<td>3 μm</td>
</tr>
<tr>
<td>1000 THz</td>
<td>0.3 μm</td>
</tr>
</tbody>
</table>

Propagation of Signal through Atmosphere Described by Radiative Transfer Theory

- Describes how radiation is altered as it travels through a homogeneous medium (here: the atmosphere)
- More specifically, describes intensity of radiation propagating a medium that simultaneously absorbs, emits, and scatters

Difference to propagation through lossy media (last lecture):
- We include microwave emission by the media itself
- This means: we include the fact that atmosphere is of high enough temperature (>200K) to emit significant microwave radiation
How does Radiative Transfer Theory Connect with Last Lecture?

- Last lecture we said:
  - Signals are reflected \((\rho)\), absorbed \((\alpha)\), or transmitted \((\Upsilon)\) through medium

- Change 1: Radiative transfer theory adds thermal emission of atmosphere itself to signal budget

- Change 2: Additional assumptions for low frequency microwave signals:
  - Homogeneous medium \(\rightarrow\) locally non-scattering \((\rho \approx 0)\) as aerosols are much smaller than wavelength
  - We also assume that medium is in thermal equilibrium \(\rightarrow\) medium is neither warming nor cooling when interacting with radiation \(\rightarrow\) Kirchhoff’s law applies (absorbed radiation equals emitted radiation: \(\alpha = \epsilon\))

- Change 3: Radiative Transfer Theory for higher frequency signals:
  - Scattering needs to be considered \((\rho \neq 0)\)

Main Question of Radiative Transfer Theory

- Given incident radiation intensity onto a slab atmosphere what is the emergent radiation on the other side?

- Intensity at arbitrary point \(s\) along path:
  - Instantaneous intensity at \(s\) = incident intensity (scaled by absorption) + accumulated emission over path (also scaled by absorption)

- Both emitted and incident radiation are subject to exponential decay due to absorption

Geometry of Radiative Transfer Theory for Passive Microwave Remote Sensing

- Arbitrary location along signal path is denoted by \(s’\)

- Incident radiation \(I_0(0)\) defined at \(s = 0\)

- At sensor radiation \(I_s(s)\) defined at sensor location

- \(\tau(s', s)\) is absorption over path from \(s’\) to \(s\)
The Radiative Transfer Equation

- We already know the atmospheric emission at point $s'$:
  \[ I'(s') \]

- As we are interested in integrative effects over a path length, we define optical depth or opacity $\tau'(s', s)$ as absorption over a path
  \[ \tau'(s', s) = \int_{s}^{s'} k'(s'') \, ds'' \]

- Opacity acts exponentially resulting in the radiative transfer equation
  \[ I(s) = I(0) \cdot \exp\left(-\tau'(0, s)\right) + \int_{0}^{s} a'(s'') \cdot \left[ b_0 F'(s'') \cdot \exp\left(-\tau'(s'', s)\right)\right] \, ds'' \]
The Radiative Transfer Equation (cont.)

- Conversion to Microwave Brightness Temperature: Radiative transfer equation is usually presented in terms of brightness temperatures
  \[ I_f = \frac{2 \pi \lambda^2}{c^2} T_b(f') \]

- If also total absorption \( \exp \left( -\tau_f(0,s) \right) \) is replaced by \( T = \exp \left( -\tau_f(0,s) \right) \) (transmissivity), radiative transfer function for fixed frequency becomes:
  \[ T_b = T_{100} F + \int_0^L k(x') \cdot T(x') \exp \left( -\tau_f(x',s) \right) dx' \]
  with real atmospheric temperature \( T(x') \) at \( x' \), atmospheric transmissity \( \tau_f \), and brightness temperature of background \( T_{100} \).

Measuring Atmospheric Gases at their Absorption/Emission Lines

![Atmospheric Absorption Spectrum](image)

- Individual atmospheric molecules have distinct narrow absorption lines → gas concentration from measuring emission at absorption lines

Example: Measuring Rain Rates from Microwave Emissions at Absorption Bands

- Measurements at 10.7GHz and 85GHz
- Sensor: TRMM (Tropical Rainfall Measuring Mission)
- Example: Typhoon Dan approaching China in Oct. 99

![Example Image](image)
Example: Global Precipitation Measurement (GPM) Mission

Faraday Rotation – Interaction of Microwave Signals with the Ionosphere

- In Ionosphere, ionization of atmospheric molecules by high-energy solar irradiance → charged medium
- Michael Faraday (1791 – 1867): In the presence of external magnetic field (geomagnetic field) polarized EM signals experience rotation of polarization vector (Faraday Rotation) when traveling through charged media

\[ \Omega = \frac{K}{r} \text{azimuth} \times TEC \]

Transmitted signal → Ionosphere → Signal at ground level

- Faraday rotation can reach up to ~25º in L-band (~25cm) and can pose severe limitations for spaceborne remote sensing in P-band (~65cm)

Mapping Ionospheric TEC using Radar Observations

- We can exploit the relationship on the previous slide to perform global mapping of ionospheric TEC from radar observations
  - Here: Spaceborne L-band SAR data from the ALOS PALSAR mission are used

\[ \text{TEC} = \frac{W}{2 \pi f r} \text{sec} \cos \theta \]

Magnetic field intensity & angle with observation direction

10º-30º geomag.
• Look at the unusual pattern in this passive microwave image acquired by NASA’s SMAP sensor:
  – This “horseshoe” pattern in the brightness temperature appeared after a 2015 flooding event in Texas and Mississippi.
  – **Q1:** What may be the cause of the lower microwave emissivity (blue)?

(Another) Think – Pair – Share

• Describe what you see: Active vs Passive microwave image of Greenland
  – The images to the left are passive (top) and active (bottom) microwave data acquired by SMAP (L-band; H polarization)
  – **Q2:** Interpret what you see and provide a physical explanation for observed patterns.

From Radiative Transfer Function to Scattering Theory

• In Radiative Transfer Theory:
  – dielectric properties in medium vary smoothly

• In Scattering Theory:
  – Interaction at boundaries between media with very different dielectric properties

We furthermore distinguish two scattering cases:

• If spatial extent of scattering medium is large relative to signal wavelength
  → Scattering from Surfaces

• If spatial extent of scattering medium is similar in scale or smaller than signal wavelength
  → Scattering from Objects
Scattering Intensity Varies with Spatial Angle

- Scattering in different spatial angles can be different
- Hence:
  - We describe scattering as an angle dependent variable
  - We calculate scattering power per unit solid angle (solid angle = steradian = Ω)
  - Steradian is the volumetric equivalent of the radian

Quantifying Scattering Effectiveness

- Effectiveness of scatterer can be quantified by scattering cross-section \( \sigma \):
  \[
  \sigma(\theta) = \frac{\text{Scattered power per unit solid angle into direction } \theta}{\text{Intensity of original incident plane wave}} \text{ [WΩ}^{-1}\text{]} 
  \]
- Note: \( \sigma(\theta) \) has dimension \( m^2 \)
- In active microwave remote sensing: interest in what is scattered back to sensor → backscattering cross-section or radar cross-section (RCS)
  \[
  \sigma = \frac{I_{\text{received}}}{I_{\text{incident}}} \frac{\lambda R^2}{4\pi} \text{ [m}^2\text{]} 
  \]
  for sensor at some distance \( R \) from the scattering interface

Parameters Influencing RCS

- Sensor Parameters
  - wavelength (e.g., penetration through canopy)
  - polarization
  - look angle
  - resolution (texture)
- Scene Parameters
  - surface roughness
  - local slope and orientation (∈ geomorphology)
  - scatterer density, e.g., biomass, leaf density
  - 3-D distribution of scatterers
  - dielectric properties \( \varepsilon_r \) of scattering material
  - soil moisture
  - vegetation status
For Some Simple Targets the RCS can be Mathematically Derived

- Example: Corner Reflectors

$$RCS = \frac{4\pi c^2}{3\lambda^2}$$

Table:

<table>
<thead>
<tr>
<th>RCS at C-band ((\lambda = 6.05 cm))</th>
<th>28.49 dB</th>
<th>23.69 dB</th>
</tr>
</thead>
</table>

How to Convert \(\sigma\) to \(\sigma_0\) and why One would do such a Thing

- RCS (\(\sigma\)) has one major disadvantage: For distributed targets, \(\sigma\) depends on size of measurement area (resolution) as seen by its unit \([m^2]\)

- To compare data from different instruments we need to normalize \(\sigma\) by the actual geometric area \(A\) of the ground surface

\[ \sigma_0 = \frac{\sigma}{A} \]

\(\sigma_0\) is called sigma naught or normalized radar cross-section (NSCR) and is unitless

Sigma Naught can be Converted to other useful Brightness Measures

Alternative Brightness Measures

- Radar brightness: \(P(\theta)\)
- \(\sigma^0(\theta)\)
- Defined in image plane
- Quantity seen in the SAR image (antenna patterns neglected)

- Gamma Naught: \(\gamma(\theta)\)
- \(\sigma^0(\theta)\)
- Equally sampled variable for Lambertian scatterers (e.g., rainforest): \(\gamma^0 = \pi \sigma^0(\theta)\)
Scientists are generally interested in quantitative measures that are referring to the ground $\sigma_0$.

For system design, values are preferred that are independent from the terrain covered $\beta_0$.

For calibration purposes, values are preferred that are equally spaced $\gamma_0$.

How Target Size Matters in Determining RCS of Single Scatterer

Scattering governed by size of target relative to signal wavelength $\lambda$.

Three regimes get distinguished (figure uses spherical scatterer as example):

- Rayleigh scattering (after Lord Rayleigh):
  - Target size: $0.1 \lambda < 2\pi r < 10 \lambda$
  - Example: Scattering of radar signal on atmospheric water droplets
  - Forward scattering dominant ($\approx$ RCS small)
  - RCS drops off with $1/r^4$

- Mie Scattering (after Gustav Mie):
  - Target size: $0.1 \lambda < 2\pi r < 10 \lambda$
  - Scattering intensity varies due to interference of signals scattered at different spots on target
  - Scattering very sensitive to small changes in $2\pi r$

- Optical scattering:
  - Target much smaller than $\lambda$ ($2\pi r \ll \lambda$ by at least factor 10)
  - Example: Scattering of radar signal on atmospheric water droplets
  - Forward scattering dominant ($\approx$ RCS small)
  - RCS drops off with $1/r^4$

Most Important Microwave Scattering Mechanisms
Definition of a Volume and Distinction from Homogeneous Media

- Composition of 3-D randomly distributed discrete scatterers
- Connection to previously discussed Homogeneous Media:
  - Scatters large enough to add scattering component
  - If density of scatterers is high and size of scatterers is low: Volume becomes homogeneous medium
  - True volumes scatter equally in all directions
- Volumes can be modeled using Radiative Transfer theory but scattering component needs to be included!

Interactions with volumes:

**RCS of Scattering from a Volume**

- **Assumptions:**
  - RCS of one scattering element is $\sigma$ and there are $N_0$ scatterers.
  - Assume the extinction coefficient of the volume is $\kappa$.
  - Volume has a thickness of $h$ causing a path length of $h\sec\theta$ ($\theta$ is incidence angle).
  - Total opacity of the layer is then $N_0 \kappa h \sec\theta$.
  - Radar illuminates area of $A$.

- **RCS $\sigma_{\text{vol}}$:**
  \[
  \sigma_{\text{vol}}(\theta) = N_0 \kappa \int_0^h N_0 \kappa \exp(-2 \kappa h \sec\theta) \, dz \quad [\text{m}^2]
  \]

- **NRCS $\sigma_{\text{NRCS}}$:**
  \[
  \sigma_{\text{NRCS}}(\theta) = \int_0^h N_0 \kappa \exp(-2 \kappa h \sec\theta) \, dz
  \]


**Example: Dependence of Volume Scattering on Volume Thickness $h$**

- C-band HH scattering from fresh water ice measured in laboratory experiment.
- Increase of SCR with ice thickness visible according to equation on previous slide.

---

**SCATTERING FROM SMOOTH SURFACES**
Scattering from Smooth Surfaces

- EM wave hitting interface at angle \( \theta_i \) is partly scattered and partly transmitted.

- For smooth surfaces, scattering forms coherent peak in specular direction \( \theta_r = \theta_i \).

- Part of signal enters medium and is slowed down causing redirection (refraction) according to Snell's Law:
  \[
  \frac{\sin \theta_i}{\sin \theta_r} = \frac{n_2}{n_1}
  \]

- Percentage of reflected energy is given by reflection coefficient \( R \) relating incident electric field \( E_i \) to reflected electric field \( E_r \):
  \[
  E_r = R \cdot E_i
  \]

Quantifying the Surface Reflection Coefficient

- \( R \) depends on relative orientation between electric field vector and surface.
- This means: \( R \) depends on the polarization of incident wave.

- For linear polarization two equations are needed to describe \( R_{VV} \) (vertical to surface in transmit and receive) and \( R_{HH} \) (horizontal to surface):

  \[
  R_{VV} = \frac{c \cdot \cos \theta_i - \sqrt{c^2 - \sin^2 \theta_i}}{c \cdot \cos \theta_i + \sqrt{c^2 - \sin^2 \theta_i}}
  \]

  \[
  R_{HH} = \frac{\cos \theta_i - \sqrt{\cos^2 \theta_i - 1}}{\cos \theta_i + \sqrt{\cos^2 \theta_i - 1}}
  \]

  \( R_{HV} = R_{VH} = 0 \)

  with \( c \) being complex dielectric constant.

- Fresnel reflection coefficients \( R_{VV} \) and \( R_{HH} \) are defined between 0 and 1.

Magnitude of Fresnel Reflection Coefficients

- One can see from both the Fresnel equations as well as the plot that \( R_{VV} = R_{HH} \) if \( \theta_i = 0^\circ \) and if \( \theta_i = 90^\circ \).
- \( R_{HH} \) is higher than \( R_{VV} \) → surface backscatter stronger in horizontal polarization.
Emission of Smooth Surfaces

- Emission $\epsilon$ can be conveniently derived from the scattering coefficient $\sigma$

- Utilizing Kirchoff’s law of $\kappa = \epsilon$ and assuming that transmissivity is zero:

  $$\epsilon_V(\theta) = 1 - |R_{VV}|$$

  $$\epsilon_H(\theta) = 1 - |R_{HH}|$$

- Apparent brightness temperature of surface is then:

  $$T_{B,V} = \epsilon_V(\theta) \cdot T$$

  $$T_{B,H} = \epsilon_H(\theta) \cdot T$$

  with $T$ being the physical temperature of the surface

SCATTERING FROM EDGES AND CORNERS

Scattering on Edges and Corners

- If $\sigma$ of both scattering surfaces is identical:
  - Scattering coefficient in HH: $R_{HH}$
  - Scattering coefficient in VV: $R_{VV}$ $\exp(j\phi)$ with $R_{VV}$ from Slide 32 and $\phi$ being a potential material dependent phase shift relative to the HH signal

  - Double-bounce scattering higher in horizontal ($R_{HH}$) than in vertical polarization ($R_{VV}$)

  - $R_{HH} = R_{VV} = 0$
SCATTERING FROM ROUGH SURFACES

Types of Scattering from Rough Surfaces

- Scattering from randomly rough surfaces
  - Agricultural fields
  - Low vegetation
  - ...

- Scattering from periodically rough surfaces (called Bragg scattering)
  - Wind driven ocean surfaces
  - Certain regularly planted crop types

When is a Surface Rough? And how do Surfaces of Varying Roughness Scatter?

- Several criteria for roughness were developed that differ in strictness including the Fraunhofer criterion (a strict criterion) states:
  \[ h_{\text{rough}} > \frac{\lambda}{2 \pi \cos \theta} \]
Schematic Analysis of Incidence Angle Dependence of Scattering

Wavelength and Surface Roughness

Scattering Processes: Bragg Scattering

Polarimetric Dependence of Bragg scattering:

- Horizontal polarization:
  \[ R_{HH} = m \cdot \frac{\cos \theta \cos \phi - \sqrt{1 - m^2}}{\cos \theta + \sqrt{1 - m^2}} \]
- Vertical polarization:
  \[ R_{VV} = m \cdot \frac{\cos \theta \cos \phi - \sqrt{1 - m^2}}{\cos \theta + \sqrt{1 - m^2}} \]
- Cross polarizations:
  \[ R_{HV} = R_{VH} = 0 \]

... where \( e \) is the dielectric constant of the surface and \( m \) depends on surface roughness.
Polarimetric Dependence of Scattering Principles

Relative scattering strength by polarization:

- Pure Surface Scattering: $|S_{vv}| > |S_{hh}| > |S_{hv}|$ or $|S_{vh}|$
- Double Bounce Scattering: $|S_{hh}| > |S_{VV}| > |S_{HV}|$ or $|S_{VH}|$
- Volume Scattering: main source of $S_{vv}$ and $S_{vh}$

What’s Next?

- From Passive to Active Microwave Sensors: Antennas, coherent sensing, and active systems!
- In preparation please read: