REVIEW OF INSAR WORKFLOW FOR A GEOPHYSICAL APPLICATION

Another Benefit of Radar:
Radar is more than just images

• With every radar acquisition, we record both **Amplitude** and **Phase** of the reflected polarized microwave signals

- Amplitude forms SAR Image
- Phase measures the range to objects on ground
- Polarization for analyzing surface types

**HH**
**VV**
**HV**
**VH**
Phase Representation

Phase is always ambiguous w.r.t. integer multiples of $2\pi$

pictorial representation of phase:
gray value
color wheel

$0$ $\pi$ $2\pi$

Interferometric SAR Measures Phase Differences Between Repeated Observations to Measure Topography and Deformation

$\Delta R = R + \Delta R$

The Concept of Differential InSAR (d-InSAR)

Interferometric Phase:

$d$-InSAR Goal: extraction of deformation signal from interferometric phase
How InSAR Works

- InSAR phase is a function of distance from satellite to ground (range)

Pass 1
Pass 2
No phase shift
No deformation

How InSAR Works

- InSAR phase is a function of distance from satellite to ground (range)

Pass 1
Pass 2
Phase shift: 120 degrees
Ground subsidence: 9 mm

InSAR Processing – Steps 1 – 7:
Find and Download Images, Co-Register & Form Interferogram

- To form interferogram, we calculate: \( I = u_1 \cdot u_2^{\ast} \) (where \( u^{\ast} \) is complex conjugate)

Example: Mt. Peulik volcano, Alaska
InSAR Processing – Step 8:
Subtraction of Flat Earth Phase

- Example:
  - ERS-2 Interferogram of Mt. Peulik volcano, Alaska

Before Flat Earth Phase Compensation
After Flat Earth Phase Compensation

InSAR Processing – Step 9 & 10:
Phase Filtering & Topographic Phase Correction using DEM

\[
\Phi_t = \phi_{def} = \frac{4\pi}{\lambda} \Delta R_{def}
\]
Filtering $\Delta \phi$ for Noise Suppression

Interpretation of Geocoded Differential Phase $\Delta \phi$

Representation of Interferometric Phase

Uplift mm
84.9
28.3

Courtesy of G. Bawden, USGS
Deformation Modeling – Concept [Mt. Peulik Example]

4 Oct. 95 - 9 Oct. 97

-156°30' -156°20' -156°10'

OBSERVED   SYNTHETIC   RESIDUAL

Best-fit source parameters:
• Spherical point source (Mogi source)
• The model source is located at a depth of 6.5 ± 0.2 km.
• The calculated volume change of magma reservoir is 0.043 ± 0.002 km³.

AN EXAMPLE OF THE USE OF INSAR IN GEOPHYSICS

Deformation Modeling Problem

deformation: what we see (InSAR)
magma dynamics: what we want to know
Deformation Modeling Problem

Estimate source characteristics from InSAR deformation data

forward model

g \cdot x = b

source parameters

displacement (vector)

inverse model

\hat{x} = G^T d

Solving for Model Parameters using Model Inversion

\begin{align*}
g \cdot x &= b \\
\text{If the covariance matrix for errors in the observation (b) is } \Sigma_{b}, \text{ then the weighted least-squares (maximum likelihood) solution for } x \text{ is:} \\
\hat{x} &= \left[ G^T \cdot \Sigma_{b}^{-1} \cdot G \right]^{-1} \cdot G^T \cdot \Sigma_{b}^{-1} \cdot b \\
\text{and the covariance matrix for the estimated vector components is:} \\
\Sigma_{\hat{x}} &= \left[ G^T \cdot \Sigma_{b}^{-1} \cdot G \right]^{-1}
\end{align*}

In the case where we assume that observation errors are independent and have equal standard deviations, \( \sigma \), we get:

\[ \Sigma_{\hat{x}} = \sigma^2 \left[ G^T \cdot G \right]^{-1} \]

- The square roots of the diagonal terms of \( \Sigma_{\hat{x}} \) are the standard errors of the estimated parameters

What is the Forward Model in Volcano Deformation?

Predicts deformation (u) caused by magma intrusion

relates magma intrusion to deformation

\[ u = f(\text{model parameters}) \]

\[ \mu \nabla^2 u = \frac{\mu}{(1-2\nu)} \left( \frac{\partial^2 u}{\partial x_1 \partial x_1} + \frac{\partial^2 u}{\partial x_2 \partial x_2} \right) = -F_i \]
What Is the Forward Model?
Simple Model: Inflating Point Source Model

- A component of deformation vector $u_\beta$ and the displacement at the free surface ($x_3 = 0$) takes the form

$$u_\beta(x_1, x_2, x_3 = 0) = C \frac{x_\beta - x^\prime_\beta}{R^2},$$

- $x^\prime_\beta$ is a source location, $C$ is a combination of material properties and source strength, and $R$ is the distance from the source to the surface location.

- $C$ is defined as follows:

$$C = \frac{\Delta P}{G} \frac{\pi}{1 - \nu} \left(1 - \nu \right),$$

- $\Delta P$ - change in pressure of magma chamber
- $\Delta V$ - change in volume of magma chamber
- $\nu$ - Poisson’s ratio
- $r_0$ - radius of the sphere
- $G$ - shear modulus of country rock

Forward Model: Inflating Point Source

Horizontal

Vertical

$\alpha/d = 0.4$

$\alpha/d = 0$ (point source)
Forward Model: Sill Model

- isotropic elastic half-space (GEOS)

Forward Model: Dike Model

Ultimate Goal of Deformation Modeling:

\[
\sum \left[ u_i(x, y) \cdot \text{los}_i(x, y) - \text{obs}_i(x, y) \right]^2
\]

- \( u_i \) is a theoretical calculation of ground surface deformation vector (i=1, 2, 3)
- \( \text{los}_i \) is the InSAR line-of-sight vector
- \( \text{obs}_i \) is the observed deformation (InSAR image)
- \((x, y)\) is the image coordinate

Non-linear inversion!!!!
Find the best-fitting Model Parameters

A Simple Approach

1. Loop through model parameters
2. calculate the residual (observed – modeled) for each set of model parameters
3. Find the set of model parameters that renders the smallest residual

A Matlab Lab for Estimating Source Parameters

xy_gridsearch.m

- defines search space for source model location – Source Depth and volume change is fixed for this experiment
- For each set of x and y coordinate parameters:
  - Run forward model to produce predicted surface deformation result [mogi2InSAR.m]
  - Calculate difference (residuals) between predicted and measured deformation
- Best fitting model parameters are those that minimize residuals

Mt. Peulik Example

Spherical point source (Mogi source)

\[ \sigma_i = \left( x'_i - x'_1, x'_2 - x'_2, R \right) = \frac{(1 - \nu) \ y - x'_3}{R} \Delta V \]

Where \( x'_1, x'_2, \) and \( x'_3 \) are horizontal locations and depth of the center of the sphere, \( R \) is the distance between the sphere and the location of observation \( (x_1, x_2, 0) \), and \( \nu \) is the Poisson’s ratio of host rock.

Best-fit source parameters:
- The model source is located at a depth of 6.5 ± 0.2 km.
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Model Inversion for Multiple Sources

- Superimposition of individual deformation sources
- Smoothing (spatial + temporal)
  - The total displacement on a given patch...
  - ...is related to that of patches adjacent to it, by a finite-difference Laplacian approximation

\[ \Delta x_i = \sum_{j \neq i} \frac{\Delta \delta x_j}{a_{ij}} \]

Example: Earthquake Modeling

Afar – triple junction
Quaternary strain localized to ~60 km long zones of fissures, aligned eruptive centers and faults - "magmatic segments"

14/9/2005 to 11/05/2005
163 earthquakes (mb <6) detected by NEIC.
Relocated by Anna Stork
3D displacements measured from radar data

Deflating Magma Chambers

Collapsed Zone along Rift

Cross section

Deformation Modelling

- 2.2 km$^3$ magma intruded along dyke (Mt St Helens 1980 1.2 km$^3$)
- 0.5 km$^3$ sourced from Dabbahu and Galbho volcanoes at North.
- Earthquakes can be responsible for < 10% of moment release.

Wright et al., Nature, 2005
What’s Next?

- This is what awaits next:
  - Thursday: Lab on InSAR Processing Using the SNAP Toolbox
  - Next week Tuesday: Project Concept Presentations