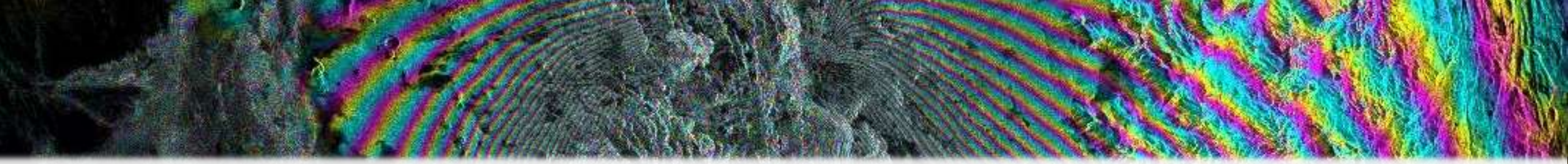


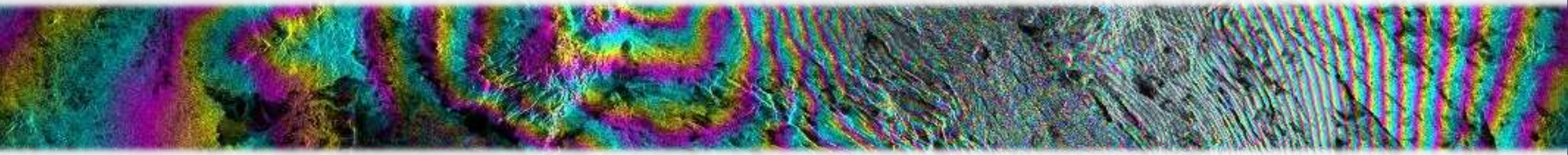
GEOS 657 – MICROWAVE REMOTE SENSING

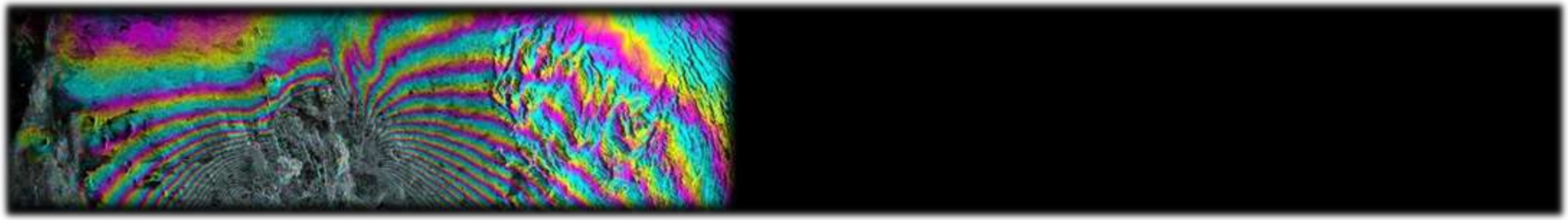
GRADUATE-LEVEL COURSE AT THE UNIVERSITY OF ALASKA FAIRBANKS

Lecturer:

Franz J Meyer, Geophysical Institute, University of Alaska Fairbanks, Fairbanks; fjmeyer@alaska.edu

Lecture 14: On the Use of InSAR in Geophysics





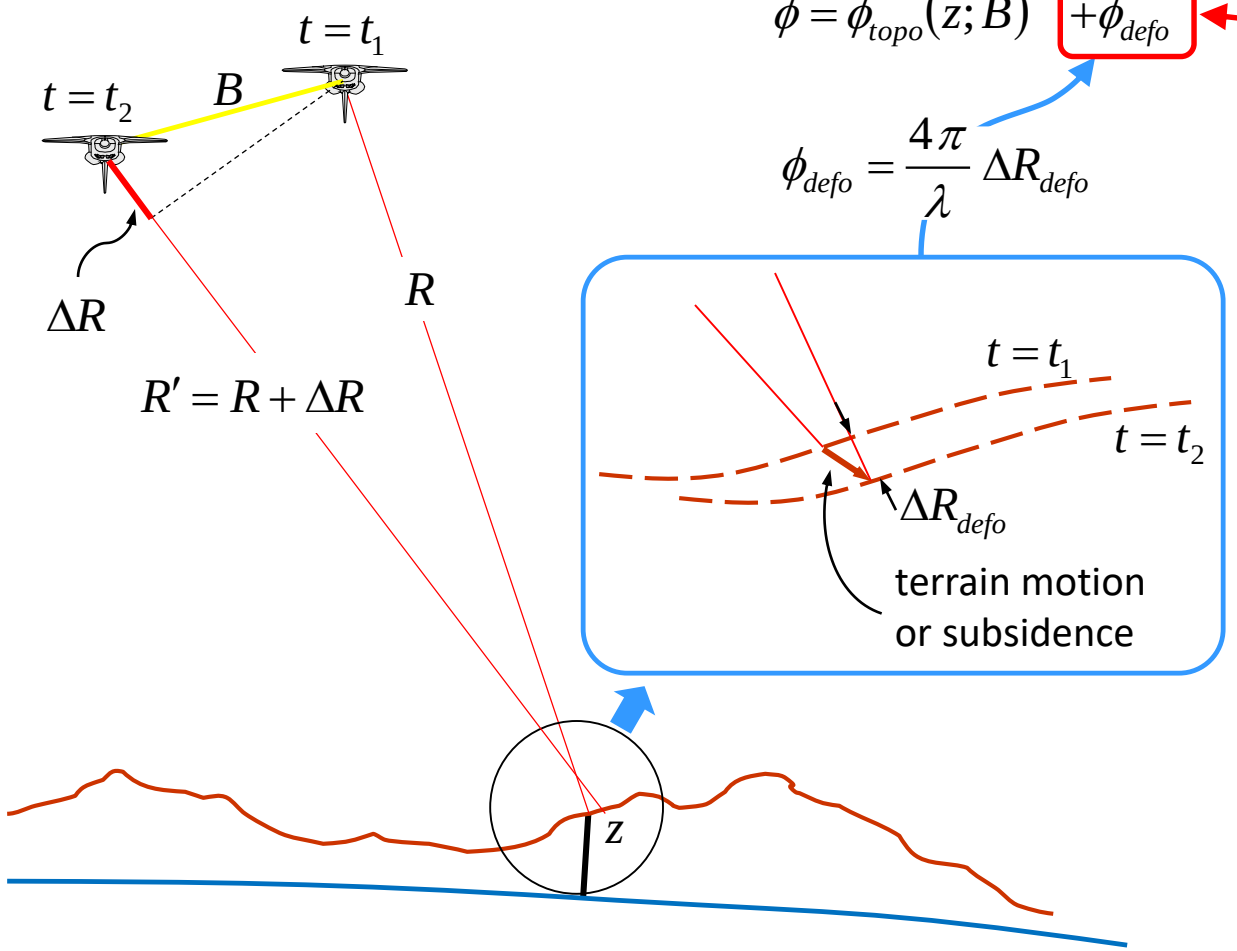
REVIEW OF INSAR WORKFLOW FOR A GEOPHYSICAL APPLICATION

The Concept of Differential InSAR (d-InSAR)

Interferometric Phase:

$$\phi = \phi_{topo}(z;B) \boxed{+\phi_{defo}}$$

$$\phi_{defo} = \frac{4\pi}{\lambda}\Delta R_{defo}$$

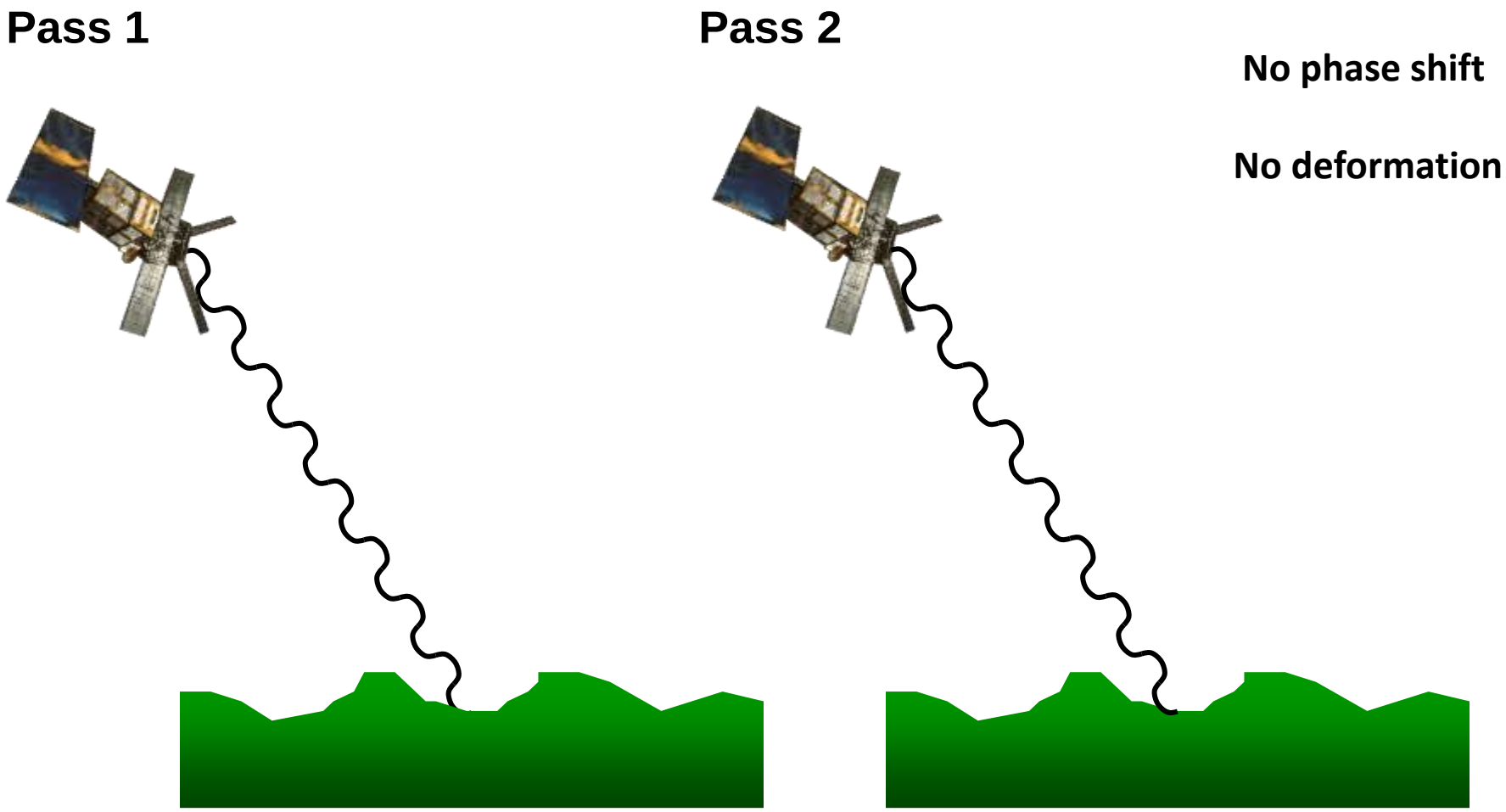


d-InSAR Goal:

extraction of deformation signal from interferometric phase

How InSAR Works

- InSAR phase is a function of distance from satellite to ground (range)



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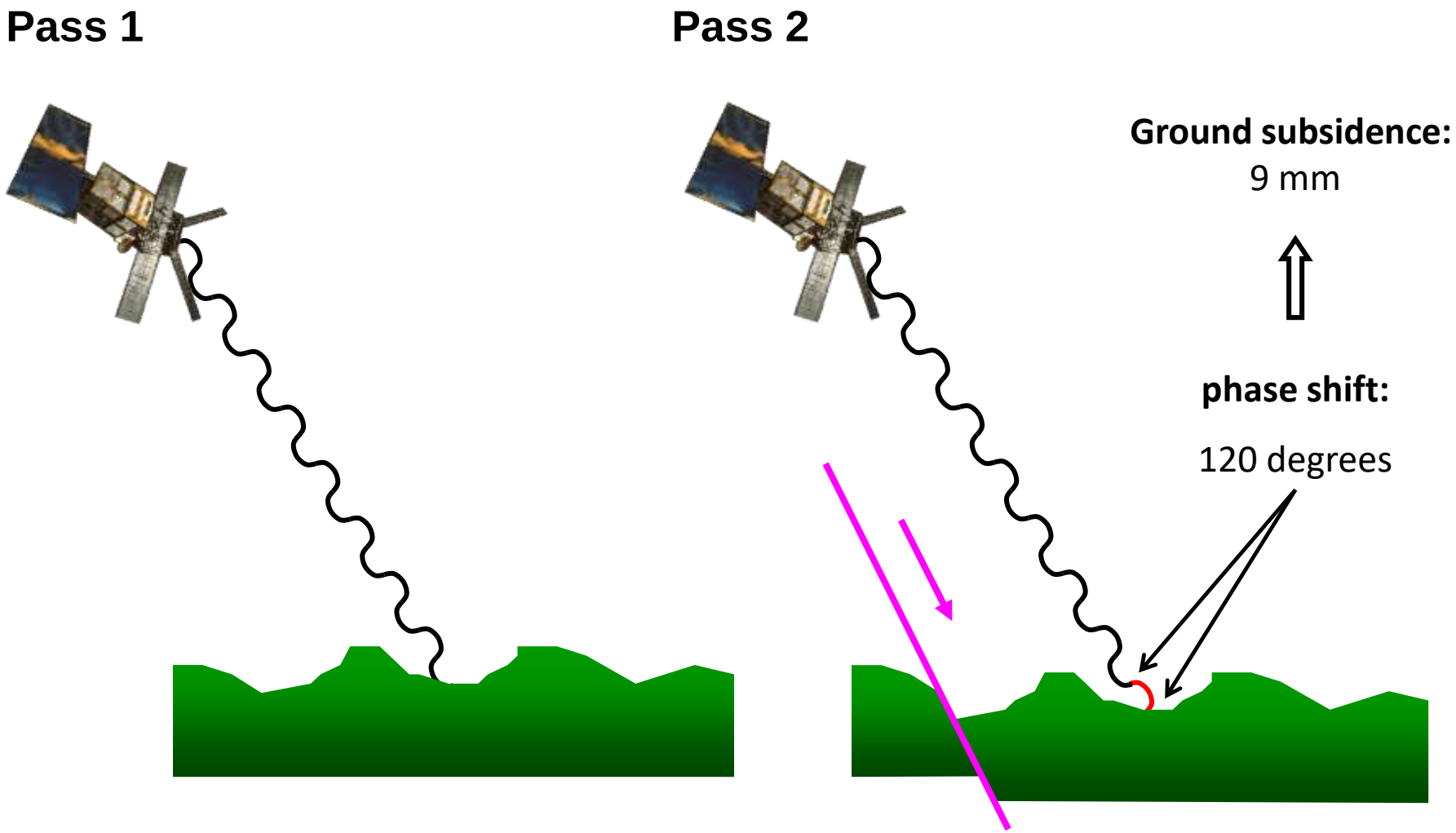
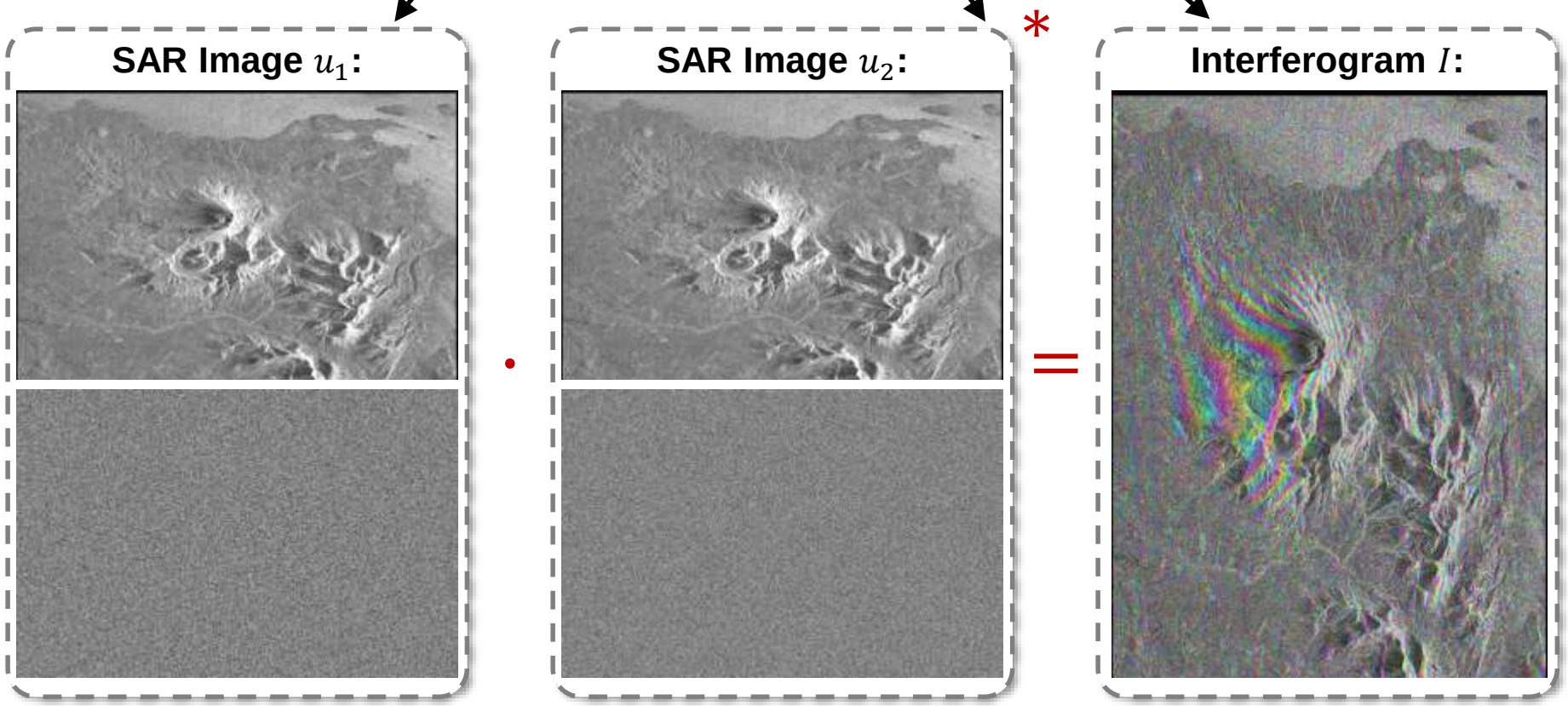


InSAR Processing – Steps 1 – 7:

Find and Download Images, Co-Register & Form Interferogram

- **To form interferogram, we calculate:** $I = u_1 \cdot u_2^*$ (where $\blacksquare^*$ is complex conjugate)

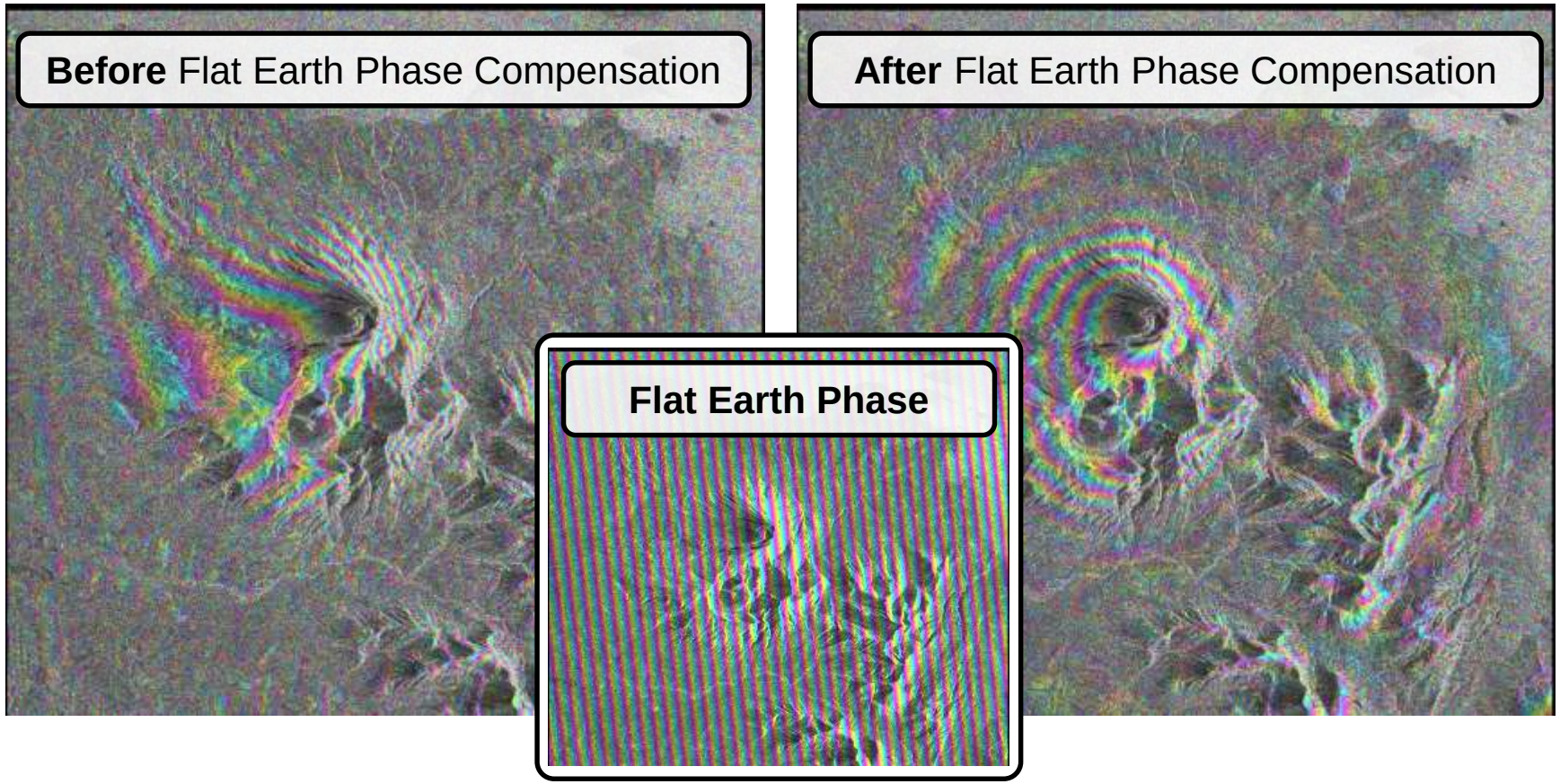


Example: Mt. Peulik volcano, Alaska

InSAR Processing – Step 8:

Subtraction of Flat Earth Phase

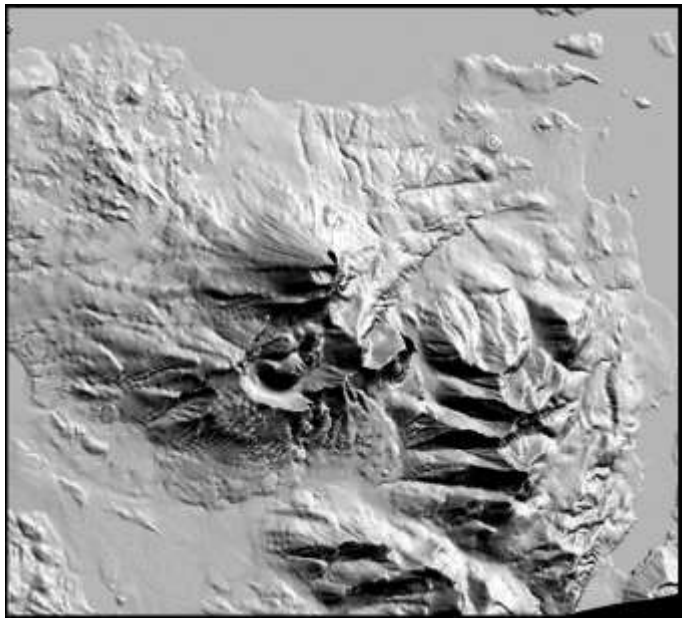
- **Example:**
  - ERS-2 Interferogram of Mt. Peulik volcano, Alaska



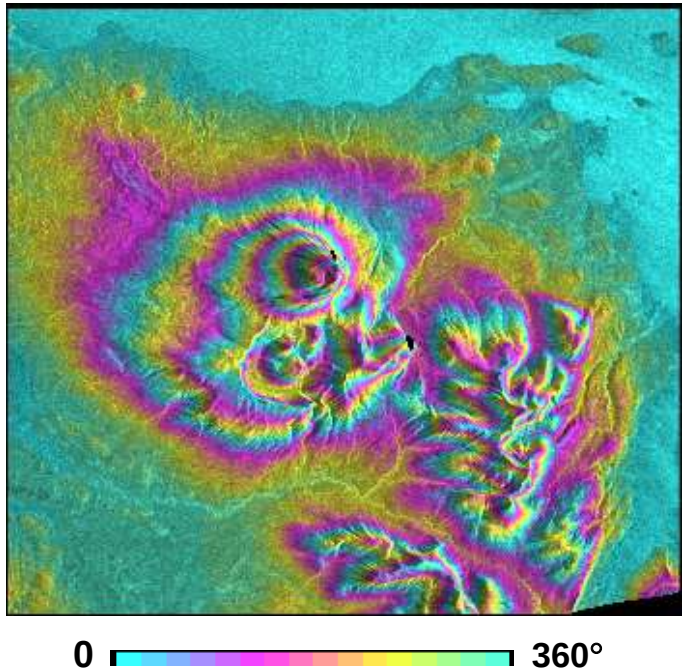
InSAR Processing – Step 9 & 10:

Phase Filtering & Topographic Phase Correction using DEM

DEM in Radar Coordinates



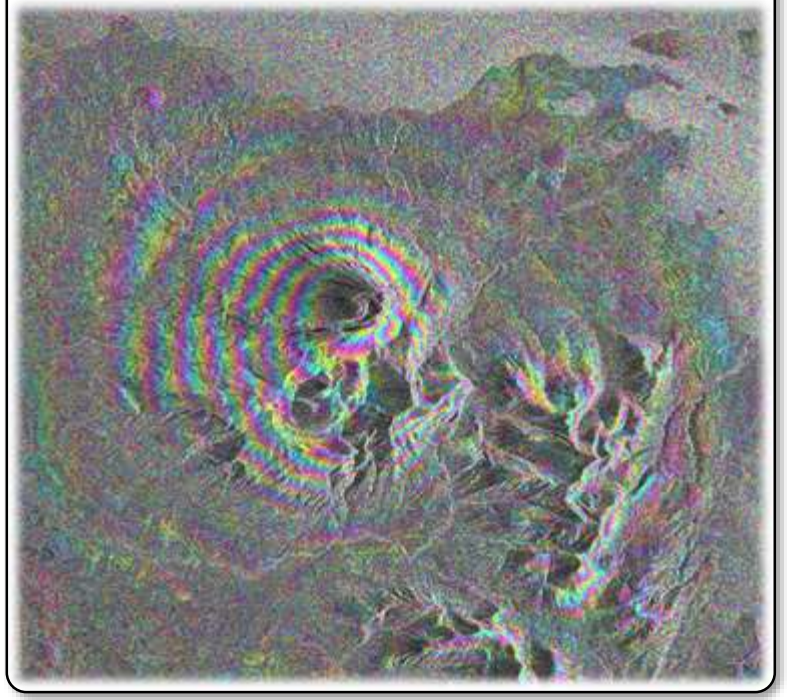
InSAR Imaging Geometry & Parameters



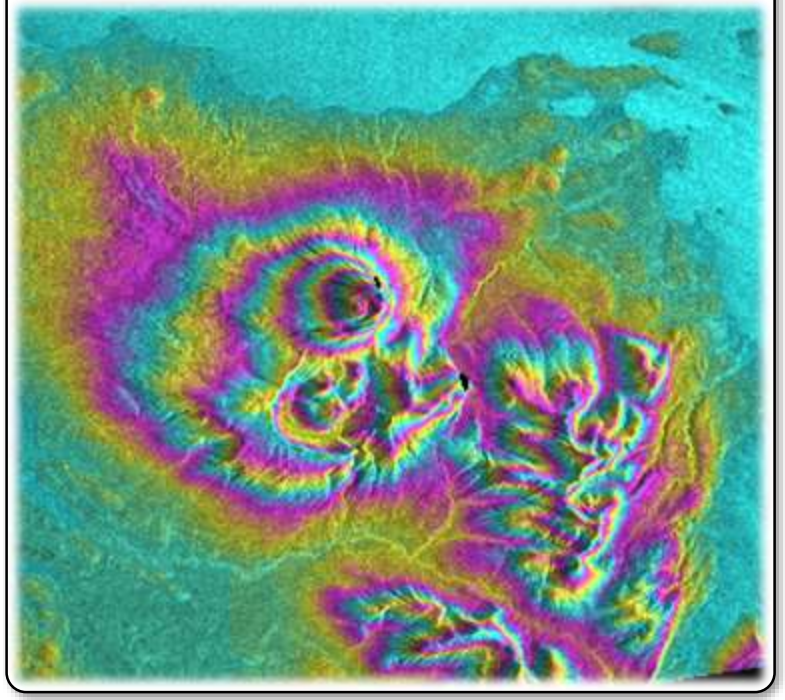
Simulated Topography-only interferogram

Topographic Phase Correction using DEM

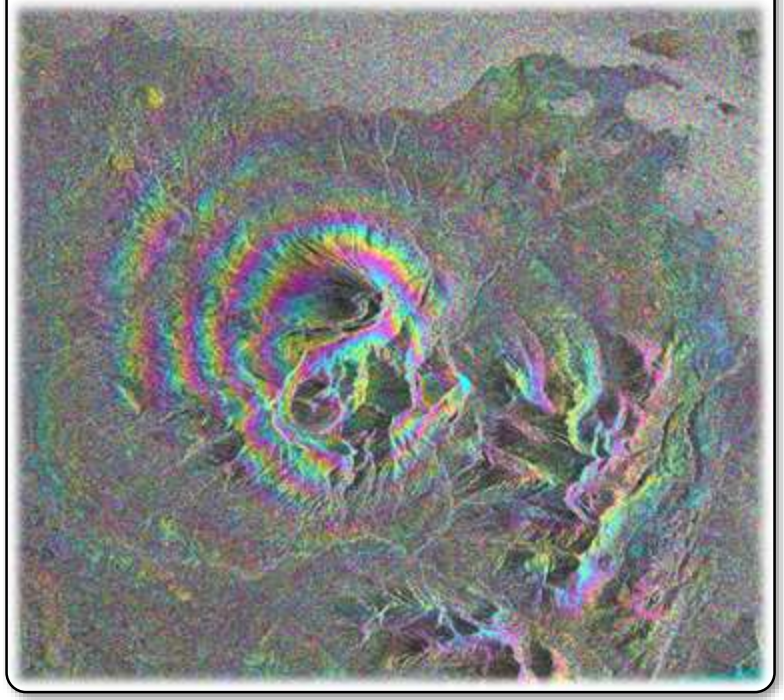
Interferogram



Topography-only interferogram

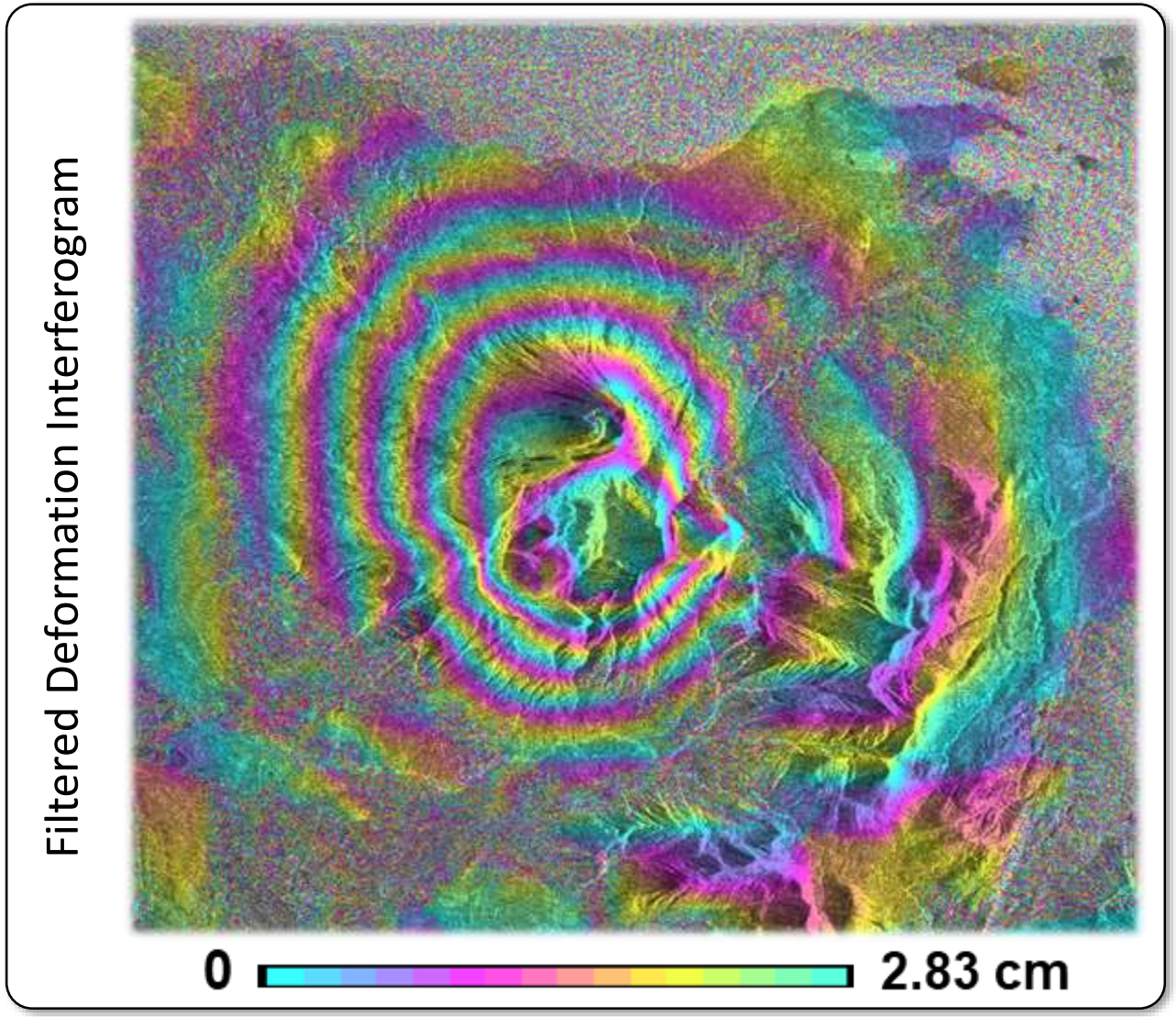


Deformation interferogram



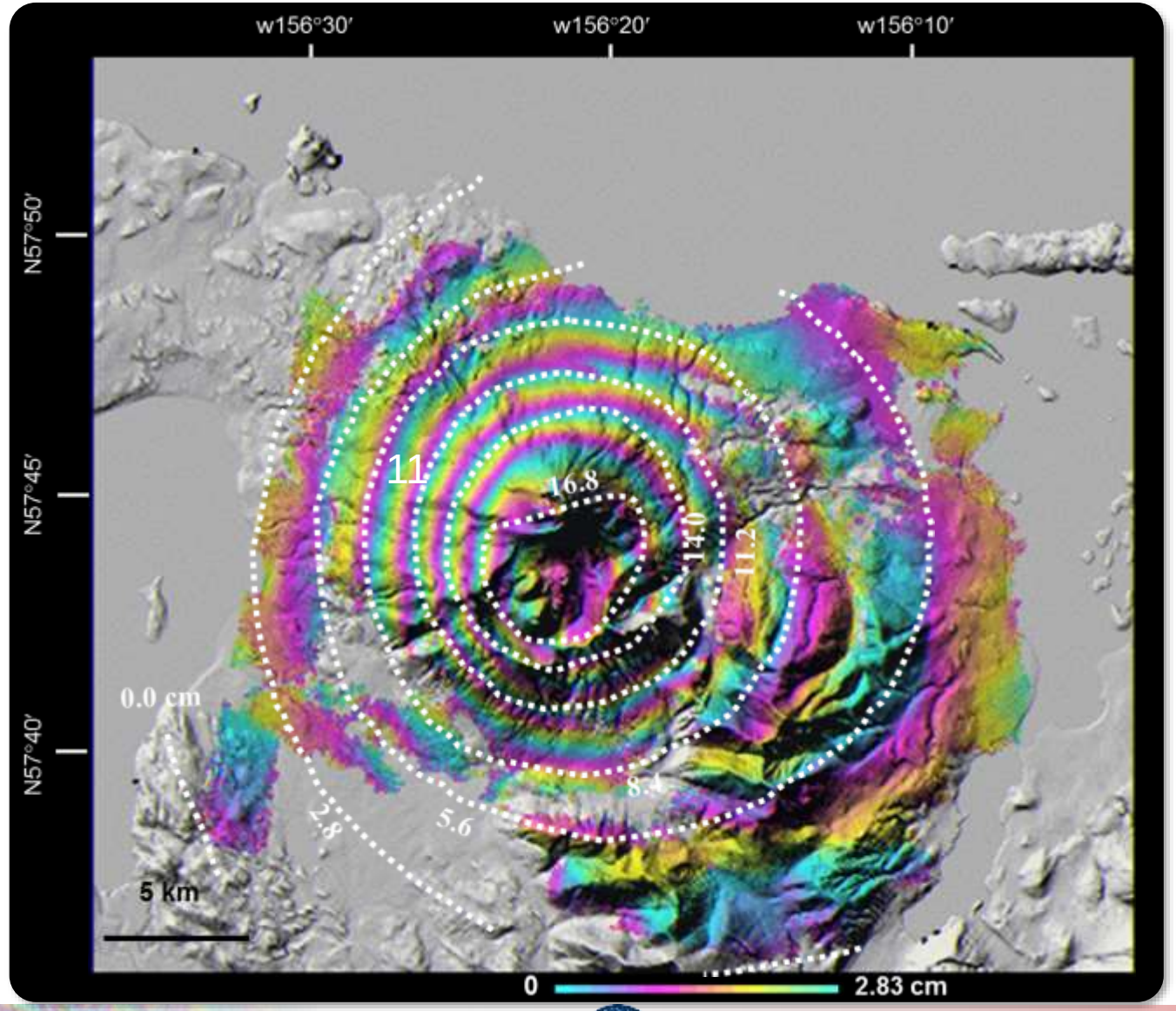
0 2.83 cm

Filtering $\Delta\phi$ for Noise Suppression



Interpretation of Geocoded Differential Phase $\Delta\phi$

Interferogram in map projection (including phase unwrapping and geocoding)



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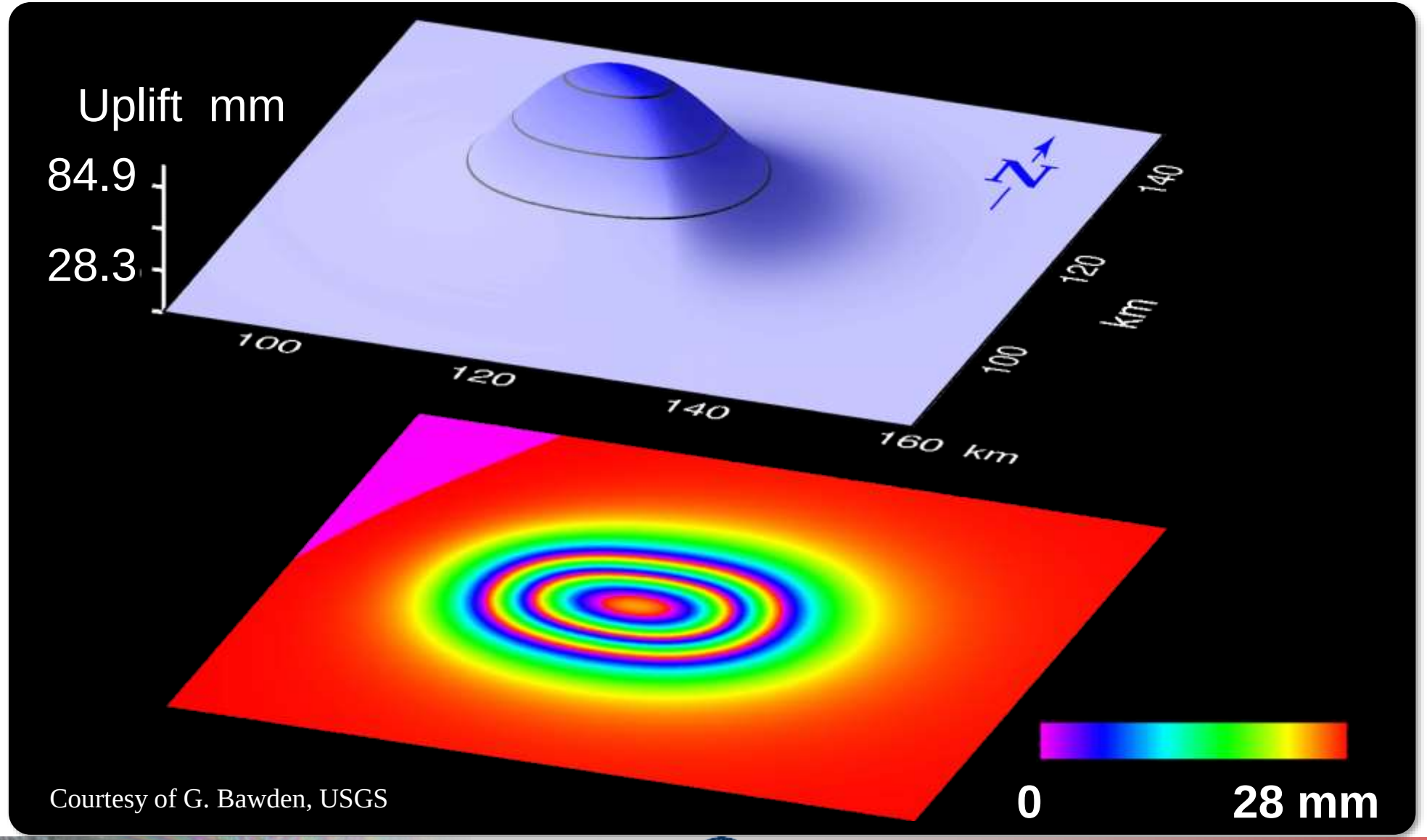
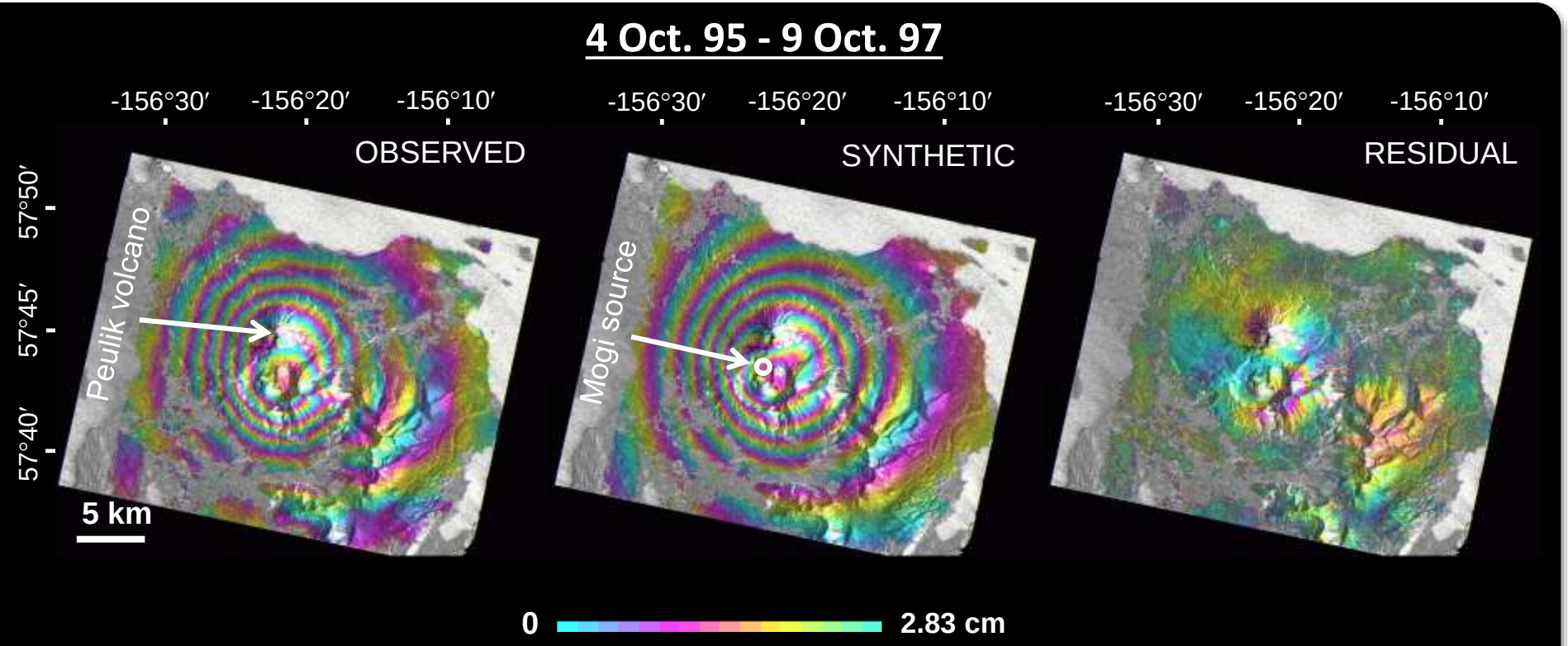


Courtesy of G. Bawden, USGS

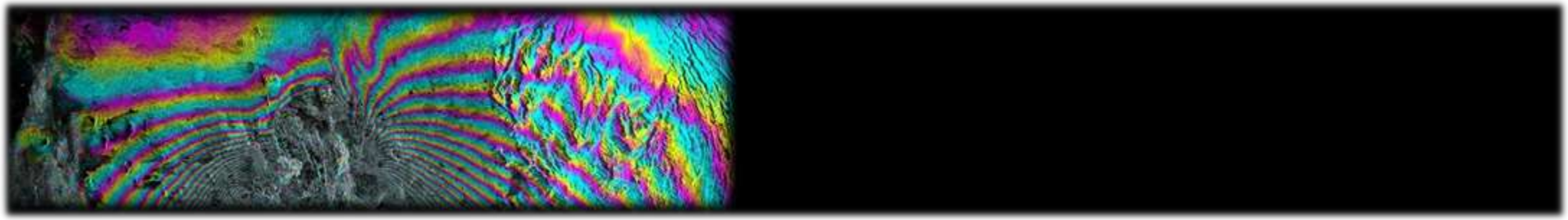
Interpret Deformation Modeling – Concept [Mt. Peulik Example]

Interferogram in map projection (including phase unwrapping and geocoding)



Best-fit source parameters:

- Spherical point source (Mogi source)
- The model source is located at a depth of 6.5 ± 0.2 km.
- The calculated volume change of magma reservoir is 0.043 ± 0.002 km^3.



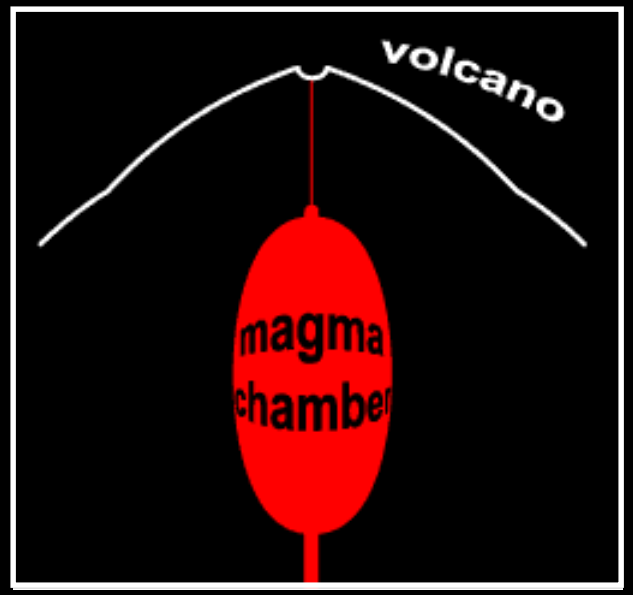
An Example of the Use of InSAR in Geophysics

The Deformation Modeling Problem

deformation:
what we see (InSAR)

?

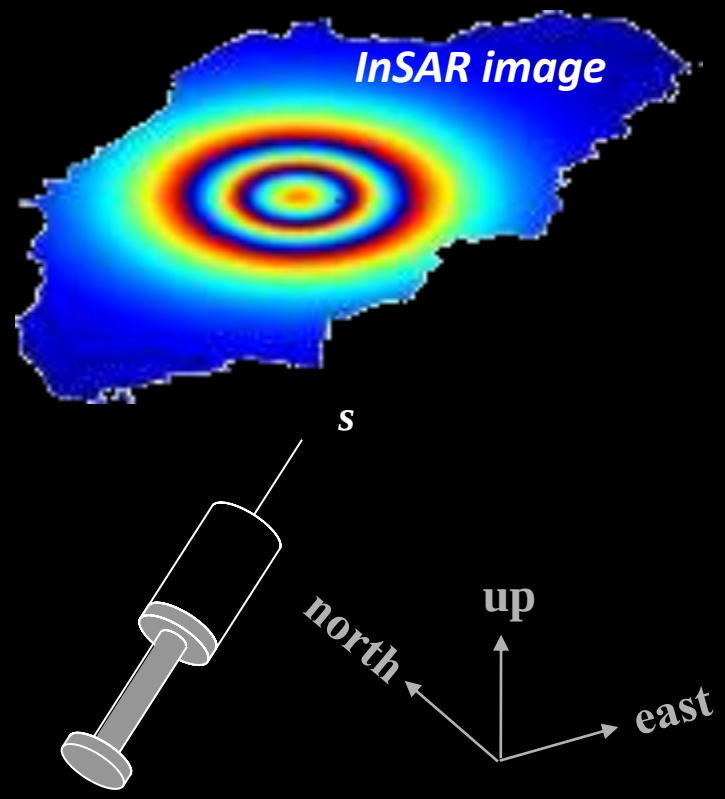
magma dynamics:
what we want to know



Magma intrusion

The Deformation Modeling Problem

Estimate source characteristics from InSAR deformation data



forward model

design matrix

$$G\,s = d$$

source parameters

displacement (vector)

inverse model

$$s = G^{inv}\,d$$

Solving for Model Parameters using Model Inversion

$$G \cdot x = b$$

- If the covariance matrix for errors in the observation (b) is Σ_b, then the weighted least-squares (maximum likelihood) solution for x is

$$\hat{x} = \left[G^T \cdot \Sigma_b^{-1} \cdot G\right]^{-1} \cdot \left[G^T \cdot \Sigma_b^{-1} \cdot b\right]$$

  and the covariance matrix for the estimated vector components is

$$\Sigma_x = \left[G^T \cdot \Sigma_b^{-1} \cdot G\right]^{-1}$$

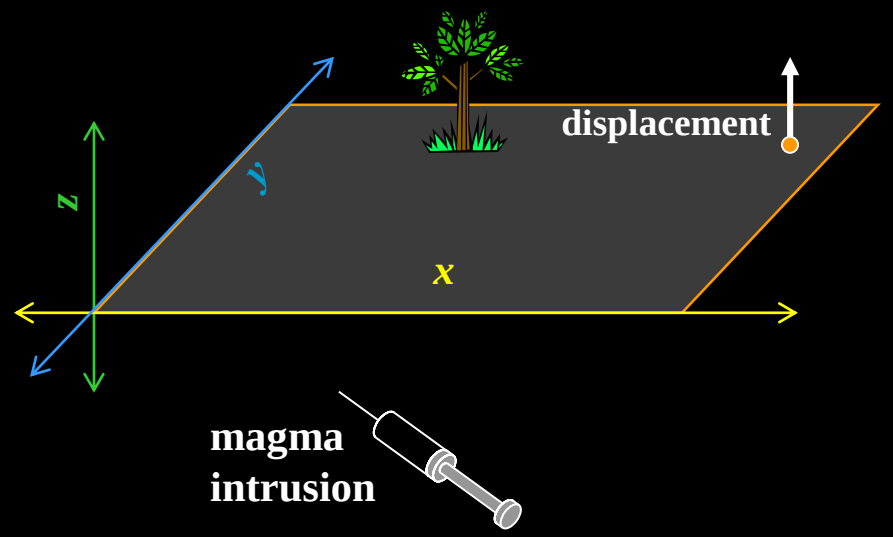
- In the case where we assume that observation errors are independent and have equal standard deviations, σ, we get

$$\Sigma_x = \sigma^2 [G^T \cdot G]^{-1}$$

  – The square roots of the diagonal terms of Σ_x are the standard errors of the estimated parameters

What is the Forward Model in Volcano Deformation?

Predicts deformation (*u*) caused by magma intrusion (relates magma intrusion to deformation)



$$\underline{u} = f(\textit{model parameters})$$

elasto-static behavior

$$\mu \nabla^2 u_i + \frac{\mu}{(1\text{-}2\upsilon)} \left[\frac{\partial^2 u_k}{\partial x_i \, \partial y_k} \right] = -F_i$$

What Is the Forward Model?

Simple Model: Inflating Point Source Model

- A component of deformation vector (u_i) and the displacement at the free surface ($x_3 = 0$) takes the form

$$u_i(x_1 - \boldsymbol{x_1'}, x_2 - \boldsymbol{x_2'}, -\boldsymbol{x_3'}) = \boldsymbol{C}\frac{x_i - \boldsymbol{x_i'}}{|R^3|}$$

  - $\boldsymbol{x_i'}$ is a source location, $\boldsymbol{C}$ is a combination of material properties and source strength, and R is the distance from the source to the surface location

- $\boldsymbol{C}$ is defined as follows:

$$\boldsymbol{C} = \Delta P(1-\nu)\frac{r_s^3}{G} = \boldsymbol{\Delta V}\frac{(1-\nu)}{\pi}$$

Unknown (target) parameters marked in **red**

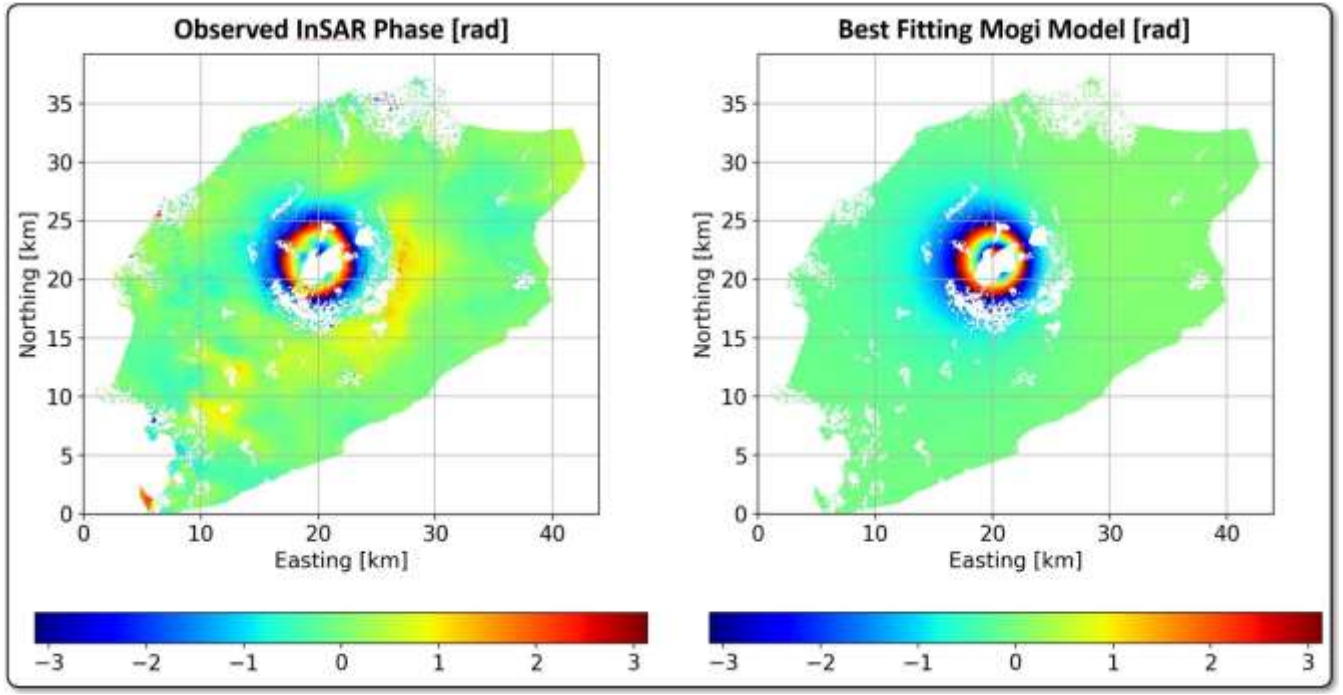
  - ΔP - change in pressure of magma chamber
  - ΔV - change in volume of magma chamber
  - ν - Poisson's ratio (material property)
  - r_s - radius of the sphere
  - G - shear modulus of country rock (material property)

Think – Pair – Share:

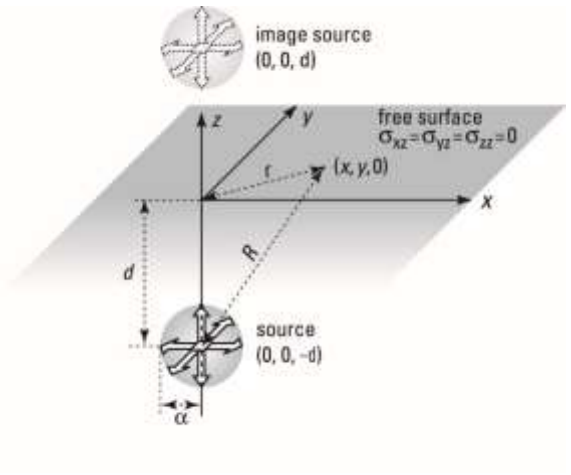
- **Limitations of Mogi Models**
  - Let's look at the Mogi model equations one more time

$$u_i(x_1 - x_1', x_2 - x_2', -x_3') = C\frac{x_i - x_i'}{|R^3|} \quad \text{with} \quad C = \Delta P(1-\nu)\frac{r_s^3}{G} = \Delta V\frac{(1-\nu)}{\pi}$$

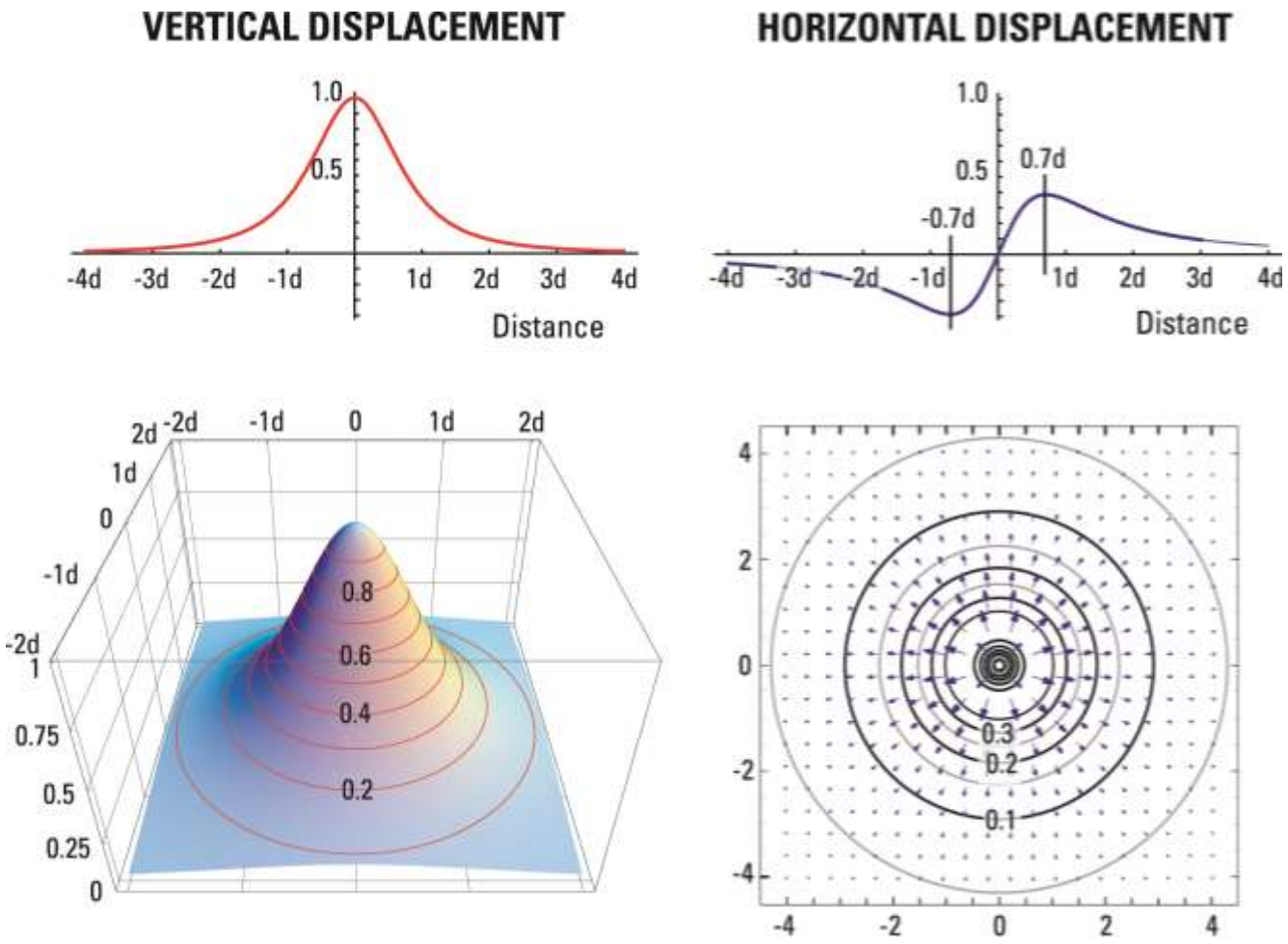
  - Activity 1: Discuss the limitations that may be brought on by how the variables ν and G are used in these equations.
  - Activity 2: Discuss the limitations that may be brought on by how the source geometry is captured in the equations.



Forward Model: Inflating Point Source

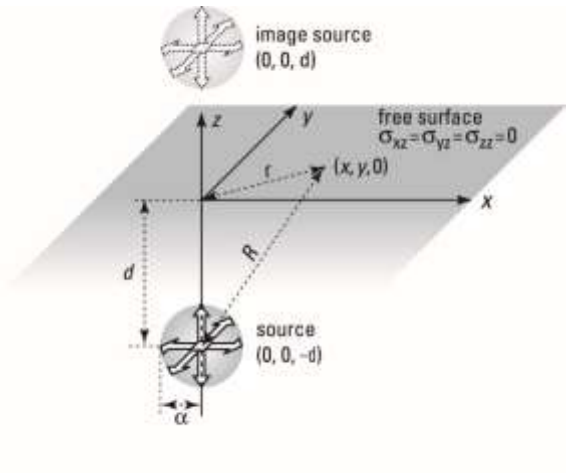


α << d

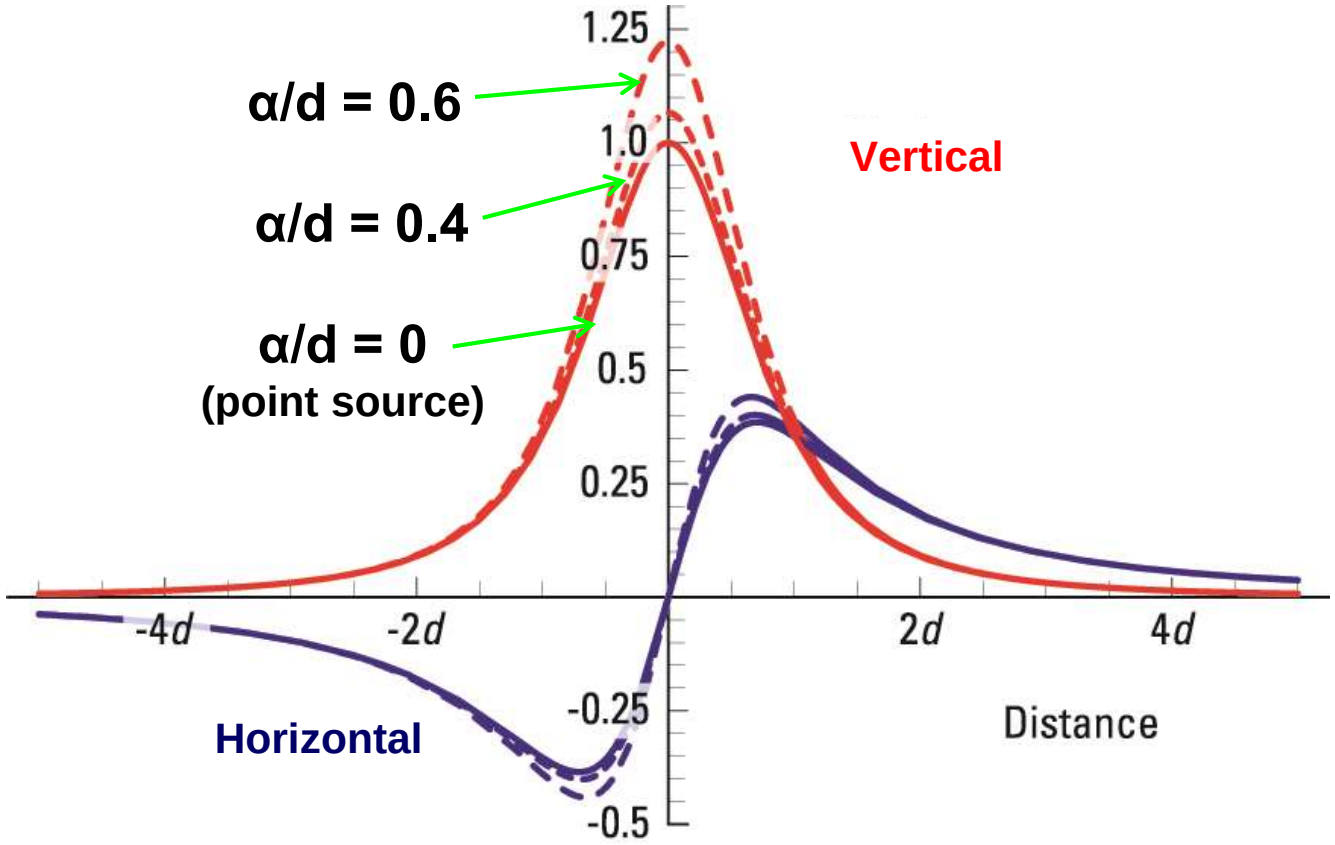


D. Dzurisin, 2007
Courtesy of M. Lisowski

Forward Model: Inflating Point Source

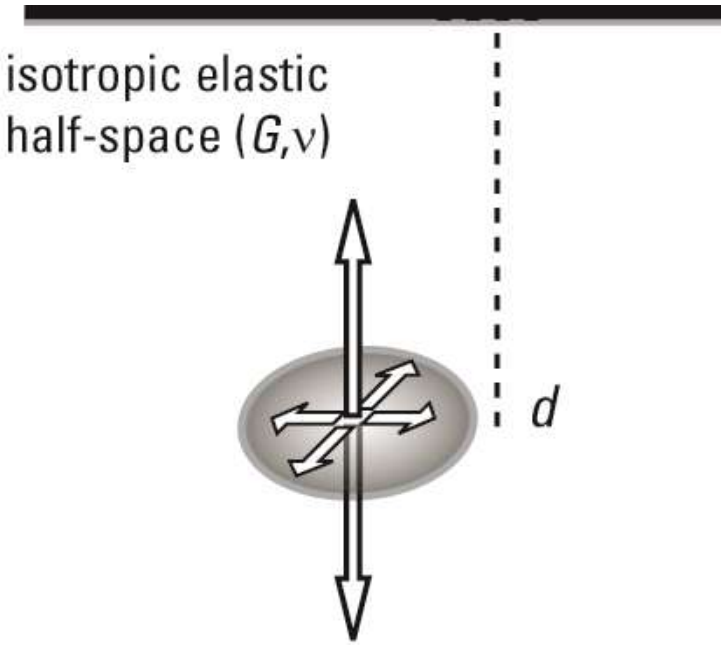


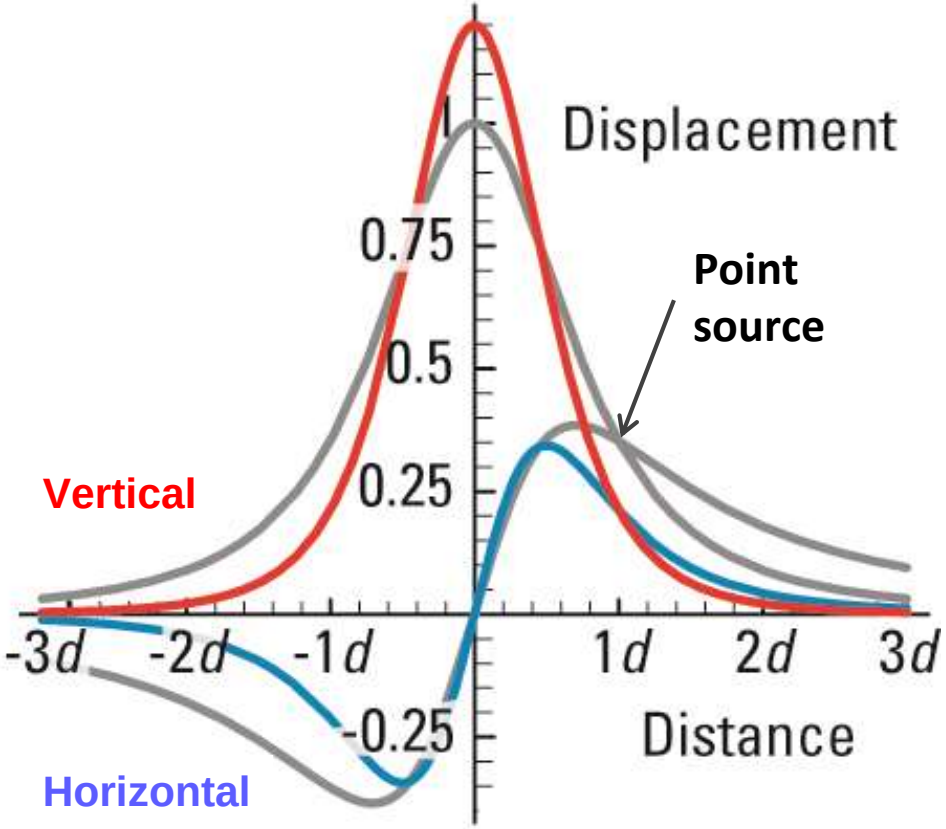
α << d



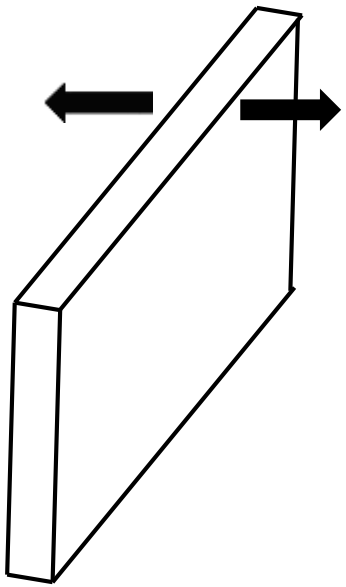
D. Dzurisin, 2007
Courtesy of M. Lisowski

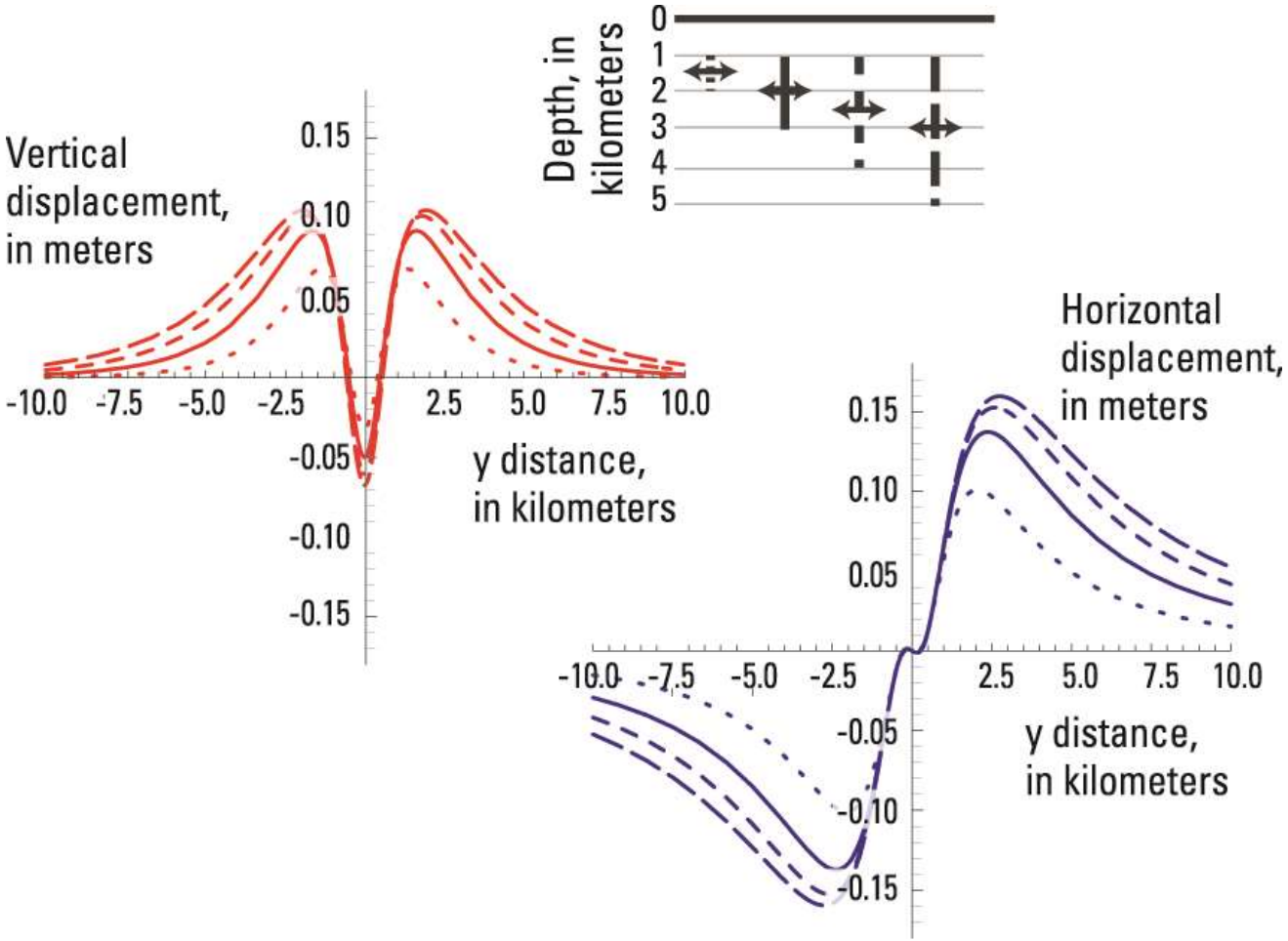
Forward Model: Sill Model





Forward Model: Dike Model





Ultimate Goal of Deformation Modeling:

Minimize

$$\sum [u_i(x,y) \bullet los_i(x,y) - obs_i(x,y)]^2$$

u_i **is a theoretical calculation of ground surface deformation vector (i=1, 2, 3)**
los_i **is the InSAR line-of-sight vector**
obs_i **is the observed deformation (InSAR image)**
(x, y) is the image coordinate

Non-linear inversion!!!!

Find the best-fitting Model Parameters

Grid Search: A Simple Approach

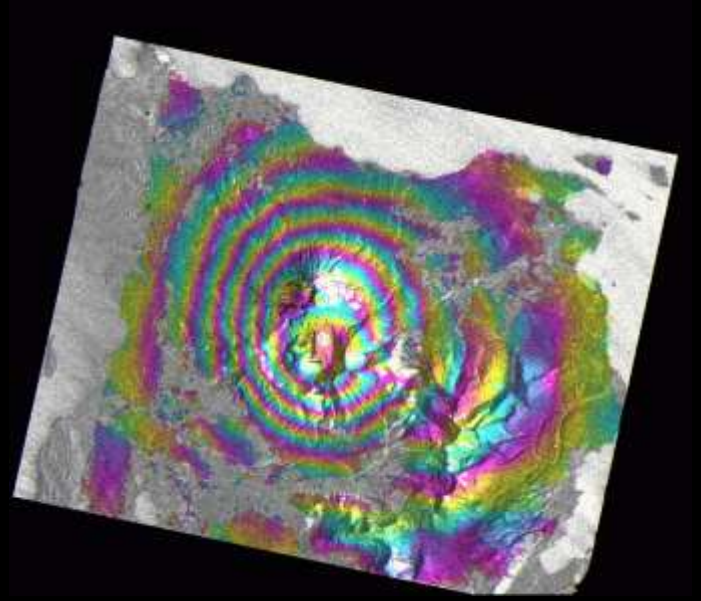
1. Loop through model parameters
2. calculate the residual (observed – modeled) for each set of model parameters
3. Find the set of model parameters that renders the smallest residual

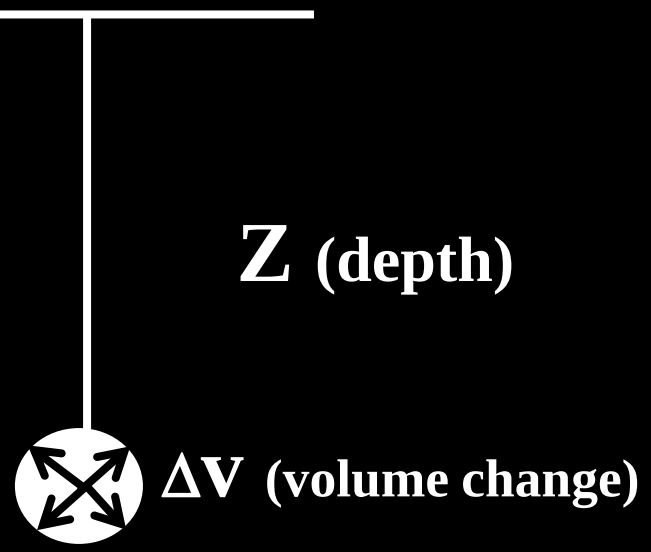
→ best-fitting model parameters

Next Week: A Jupyter Notebook Lab for Estimating Source Parameters

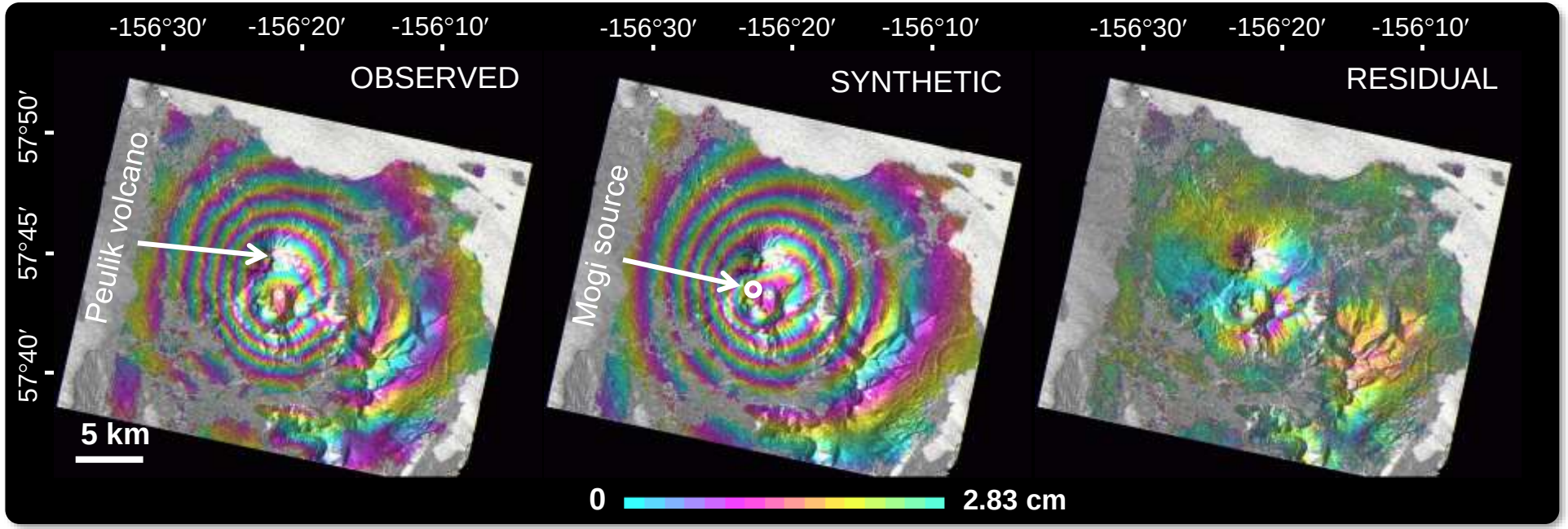
What we will do in the lab:

- We will define a search space for source model location
- We will assume that source depth and magma volume change are known and fixed
- **For each set of x and y coordinate parameters:**
  - We will run a forward model to produce *predicted surface deformation* results
  - Calculate difference (residuals) between predicted and measured deformation
- **Best fitting model parameters are those that minimize residuals between observations & model prediction**





Mt. Peulik Example



- Spherical Point Source Model (Mogi Source)

$$u_i(x_1 - \boldsymbol{x_1'}, x_2 - \boldsymbol{x_2'}, -\boldsymbol{x_3'}) = \boldsymbol{C}\frac{x_i - \boldsymbol{x_i'}}{|R^3|}$$

Where $\boldsymbol{x_i'}$ is source location, $\boldsymbol{C}$ is a combination of material properties and source strength, and R is the distance from the source to the surface location

- Best fit Source parameters:
  - Depth: $6.5 \pm 0.2km$; Volume change: $0.043 \pm 0.002km^3$

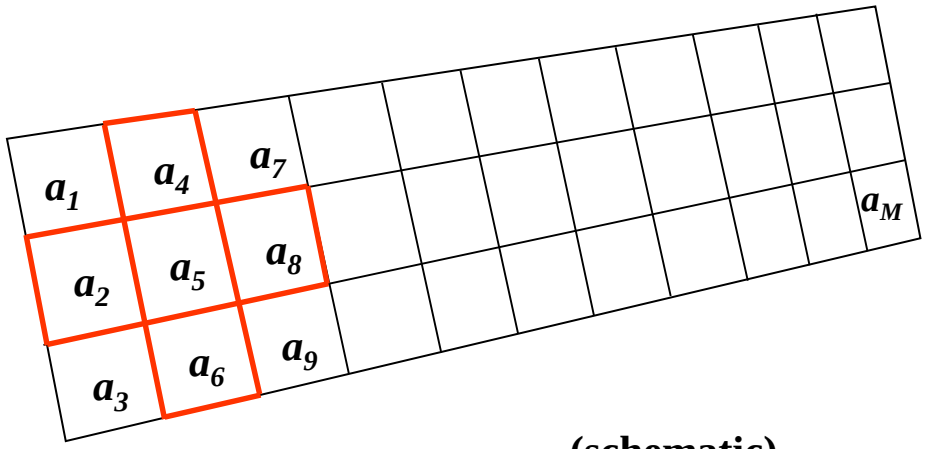
Model Inversion for Multiple Sources

- **Superimposition of individual deformation sources**

- Smoothing (spatial + temporal)
  - The total displacement on a given patch…
  - …is related to that of patches adjacent to it, by a difference Laplacian approximation

$$(a_2 - a_5) - (a_5 - a_8) + (a_4 - a_5) - (a_5 - a_6) = 0$$

$$a_2 + a_4 - 4a_5 + a_6 + a_8 = 0$$



(schematic)

What's Next?

- **This is what awaits next:**
  - **Next week Tuesday**: Lab on Mogi source inversion from InSAR