Lecture 14: On the Use of InSAR in Geophysics
REVIEW OF INSAR WORKFLOW FOR A GEOPHYSICAL APPLICATION
The Concept of Differential InSAR (d-InSAR)

Interferometric Phase:
\[ \phi = \phi_{\text{topo}}(z; B) + \phi_{\text{defo}} \]
\[ \phi_{\text{defo}} = \frac{4\pi}{\lambda} \Delta R_{\text{defo}} \]

d-InSAR Goal:
extraction of deformation signal from interferometric phase
How InSAR Works

- InSAR phase is a function of distance from satellite to ground (range)

Pass 1

Pass 2

No phase shift

No deformation
How InSAR Works

- InSAR phase is a function of distance from satellite to ground (range)

Pass 1

Pass 2

Ground subsidence: 9 mm

Phase shift: 120 degrees
InSAR Processing – Steps 1 – 7:
Find and Download Images, Co-Register & Form Interferogram

- To form interferogram, we calculate: $I = u_1 \cdot u_2^*$ (where $\cdot$ is complex conjugate)

Example: Mt. Peulik volcano, Alaska
InSAR Processing – Step 8:
Subtraction of Flat Earth Phase

• Example:
  – ERS-2 Interferogram of Mt. Peulik volcano, Alaska
InSAR Processing – Step 9 & 10:
Phase Filtering & Topographic Phase Correction using DEM

DEM in Radar Coordinates

InSAR Imaging
Geometry & Parameters

Simulated Topography-only interferogram
Topographic Phase Correction using DEM

Interferogram

Topography-only interferogram

Deformation interferogram

\[ 0 = 2.83 \text{ cm} \]
Filtering $\Delta \phi$ for Noise Suppression
Interpretation of Geocoded Differential Phase $\Delta \phi$

Interferogram in map projection (including phase unwrapping and geocoding)
Interpretation of Geocoded Differential Phase $\Delta \phi$

Interferogram in map projection (including phase unwrapping and geocoding)

 Courtesy of G. Bawden, USGS
Interpret Deformation Modeling – Concept [Mt. Peulik Example]

Interferogram in map projection (including phase unwrapping and geocoding)

Best-fit source parameters:
- Spherical point source (Mogi source)
- The model source is located at a depth of 6.5 ± 0.2 km.
- The calculated volume change of magma reservoir is 0.043 ± 0.002 km³.
An Example of the Use of InSAR in Geophysics
The Deformation Modeling Problem

deformation:  
what we see (InSAR)

magma dynamics:  
what we want to know
The Deformation Modeling Problem

Estimate source characteristics from InSAR deformation data

forward model

design matrix

G \mathbf{s} = \mathbf{d}

source parameters

displacement (vector)

inverse model

s = G^{-1} \mathbf{d}
Solving for Model Parameters using Model Inversion

\[ G \cdot x = b \]

- If the covariance matrix for errors in the observation \( (b) \) is \( \Sigma_b \), then the weighted least-squares (maximum likelihood) solution for \( x \) is

\[ \hat{x} = \left( G^T \cdot \Sigma_b^{-1} \cdot G \right)^{-1} \cdot \left( G^T \cdot \Sigma_b^{-1} \cdot b \right) \]

and the covariance matrix for the estimated vector components is

\[ \Sigma_x = \left( G^T \cdot \Sigma_b^{-1} \cdot G \right)^{-1} \]

- In the case where we assume that observation errors are independent and have equal standard deviations, \( \sigma \), we get

\[ \Sigma_x = \sigma^2 [G^T \cdot G]^{-1} \]

- The square roots of the diagonal terms of \( \Sigma_x \) are the standard errors of the estimated parameters.
What is the Forward Model in Volcano Deformation?

Predicts deformation ($u$) caused by magma intrusion (relates magma intrusion to deformation)

$$u = f(\text{model parameters})$$
What Is the Forward Model?

Simple Model: Inflating Point Source Model

- A component of deformation vector $(u_i)$ and the displacement at the free surface $(x_3 = 0)$ takes the form

$$u_i(x_1 - x_1', x_2 - x_2', -x_3') = C \frac{x_i - x_i'}{|R^3|}$$

- $x_i'$ is a source location, $C$ is a combination of material properties and source strength, and $R$ is the distance from the source to the surface location

- $C$ is defined as follows:

$$C = \Delta P (1 - \nu) \frac{r_s^3}{G} = \Delta V \frac{(1 - \nu)}{\pi}$$

- $\Delta P$ - change in pressure of magma chamber
- $\Delta V$ - change in volume of magma chamber
- $\nu$ - Poisson’s ratio (material property)
- $r_s$ - radius of the sphere
- $G$ - shear modulus of country rock (material property)
Think – Pair – Share:

- **Limitations of Mogi Models**
  
  - Let’s look at the Mogi model equations one more time

  \[ u_i(x_1 - x'_1, x_2 - x'_2, -x'_3) = C \frac{x_i - x'_i}{|R^3|} \]

  with

  \[ C = \Delta P (1 - \nu) \frac{r^3}{G} = \Delta V \frac{(1-\nu)}{\pi} \]

- **Activity 1**: Discuss the limitations that may be brought on by how the variables \( \nu \) and \( G \) are used in these equations.

- **Activity 2**: Discuss the limitations that may be brought on by how the source geometry is captured in the equations.
Forward Model: Inflating Point Source

\[ \alpha \ll d \]

D. Dzurisin, 2007

Courtesy of M. Lisowski
Forward Model: Inflating Point Source

α << d

D. Dzurisin, 2007
Courtesy of M. Lisowski
Forward Model: Sill Model

isotropic elastic half-space \((G, \nu)\)

\[ \text{Point source} \]

\[ \text{Vertical} \]

\[ \text{Horizontal} \]

\[ d \]
Forward Model: Dike Model
Ultimate Goal of Deformation Modeling:

Minimize

$$\sum [u_i(x, y) \cdot los_i(x, y) - obs_i(x, y)]^2$$

$u_i$ is a theoretical calculation of ground surface deformation vector (i=1, 2, 3)

$los_i$ is the InSAR line-of-sight vector

$obs_i$ is the observed deformation (InSAR image)

$(x, y)$ is the image coordinate

Non-linear inversion!!!!
Find the best-fitting Model Parameters

Grid Search: A Simple Approach

1. Loop through model parameters
2. calculate the residual (observed – modeled) for each set of model parameters
3. Find the set of model parameters that renders the smallest residual

→ best-fitting model parameters
Next Week: A Jupyter Notebook Lab for Estimating Source Parameters

What we will do in the lab:

- We will define a search space for source model location
- We will assume that source depth and magma volume change are known and fixed

- For each set of x and y coordinate parameters:
  - We will run a forward model to produce *predicted surface deformation* results
  - Calculate difference (residuals) between predicted and measured deformation

- Best fitting model parameters are those that minimize residuals between observations & model prediction
Mt. Peulik Example

- Spherical Point Source Model (Mogi Source)

\[ u_i(x_1 - x'_1, x_2 - x'_2, -x'_3) = C \frac{x_i - x'_i}{R^3} \]

Where \( x'_i \) is source location, \( C \) is a combination of material properties and source strength, and \( R \) is the distance from the source to the surface location.

- Best fit Source parameters:
  - Depth: 6.5 ± 0.2 km; Volume change: 0.043 ± 0.002 km³
Model Inversion for Multiple Sources

- Superimposition of individual deformation sources
  - The total displacement on a given patch...
  - ...is related to that of patches adjacent to it, by a difference Laplacian approximation

\[
(a_2-a_5)-(a_5-a_8)+(a_4-a_5)-(a_5-a_6) = 0 \\
\therefore a_2 + a_4 - 4a_5 + a_6 + a_8 = 0
\]

(schematic)
What’s Next?

• This is what awaits next:
  – **Next week Tuesday**: Lab on Mogi source inversion from InSAR