GEOS 657 – MICROWAVE REMOTE SENSING

GRADUATE-LEVEL COURSE AT THE UNIVERSITY OF ALASKA FAIRBANKS

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Lecture 14: On the Use of InSAR in Geophysics

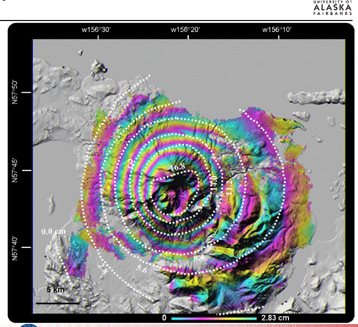
UAF Course GEOS 657

The Use of InSAR in Geophysics

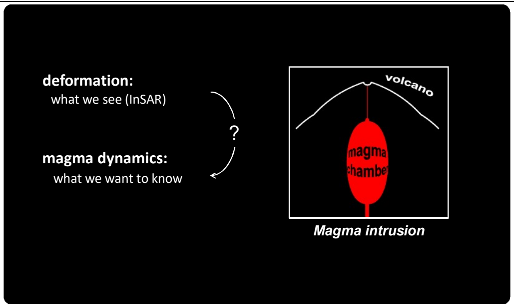
InSAR Helps us Measure Surface Displacement

Example: Mt. Peulik volcano, Alaska

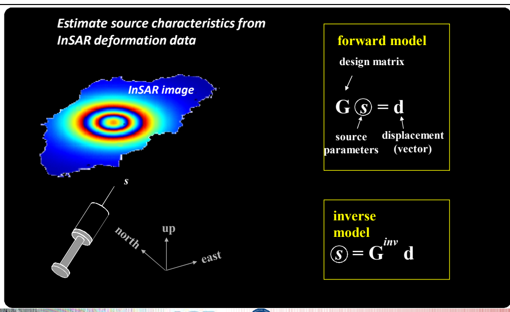
- InSAR gives us a geodetic measurement in the sense that it provides information about a pixels location in the world and about its movement.
- InSAR does not tell us WHY the surface is moving in the observed way.
- To determine the geophysical cause of the observed motion, we need to combine our InSAR observables with geophysical models



The Deformation Modeling Problem



The Deformation Modeling Problem



Solving for Model Parameters using Model Inversion

$$G \cdot x = b$$

- If the covariance matrix for errors in the observation (b) is Σ_b, then the weighted least-squares (maximum likelihood) solution for x is

$$\hat{x} = \left[G^T \cdot \Sigma_b^{-1} \cdot G\right]^{-1} \cdot \left[G^T \cdot \Sigma_b^{-1} \cdot b\right]$$

and the covariance matrix for the estimated vector components is

$$\Sigma_x = \left[G^T \cdot \Sigma_b^{-1} \cdot G\right]^{-1}$$

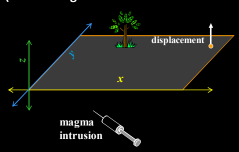
- In the case where we assume that observation errors are independent and have equal standard deviations, σ, we get

$$\Sigma_x = \sigma^2 \left[G^T \cdot G\right]^{-1}$$

  - The square roots of the diagonal terms of Σ_x are the standard errors of the estimated parameters

What is the Forward Model in Volcano Deformation?

Predicts deformation (*u*) caused by magma intrusion (relates magma intrusion to deformation)



$\underline{u} = f(\text{model parameters})$

elasto-static behavior

$$\mu \nabla^2 u_i + \frac{\mu}{(1-2\upsilon)}\left[\frac{\partial^2 u_k}{\partial x_i \partial y_k}\right] = -F_i$$

What Is the Forward Model?

Simple Model: Inflating Point Source Model

- A component of deformation vector (u_i) and the displacement at the free surface ($x_3 = 0$) takes the form

$$u_i(x_1 - x'_1, x_2 - x'_2, -x'_3) = C\frac{x_i - x'_i}{|R^3|}$$

  - x'_i is a source location, C is a combination of material properties and source strength, and R is the distance from the source to the surface location

- C is defined as follows:

$$C = \Delta P(1-\nu)\frac{r_s^3}{G} = \Delta V\frac{(1-\nu)}{\pi}$$

Unknown (target) parameters marked in red

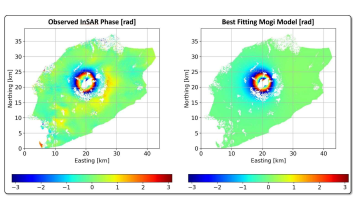
  - ΔP - change in pressure of magma chamber
  - ΔV - change in volume of magma chamber
  - ν - Poisson's ratio (material property)
  - r_s - radius of the sphere
  - G - shear modulus of country rock (material property)

Think – Pair – Share:

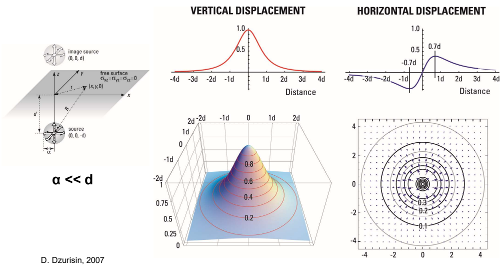
- **Limitations of Mogi Models**
  - Let's look at the Mogi model equations one more time

$$u_i(x_1 - x'_1, x_2 - x'_2, -x'_3) = C\frac{x_i - x'_i}{|R^3|} \quad \text{with} \quad C = \Delta P(1-\nu)\frac{r_s^3}{G} = \Delta V\frac{(1-\nu)}{\pi}$$

  - Activity 1: Discuss the limitations that may be brought on by how the variables ν and G are used in these equations.
  - Activity 2: Discuss the limitations that may be brought on by how the source geometry is captured in the equations.

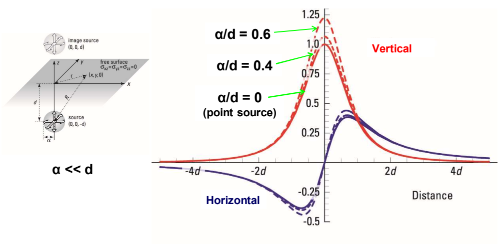


Forward Model: Inflating Point Source



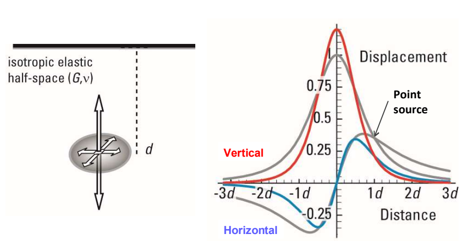
D. Dzurisin, 2007
Courtesy of M. Lisowski

Forward Model: Inflating Point Source

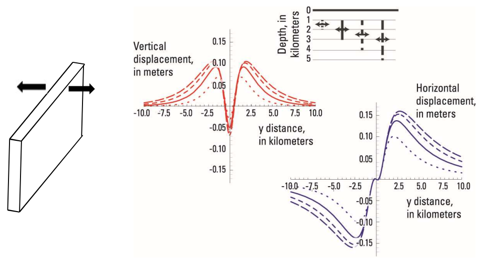


D. Dzurisin, 2007
Courtesy of M. Lisowski

Forward Model: Sill Model



Forward Model: Dike Model



Ultimate Goal of Deformation Modeling:

Minimize

$$\sum[u_i(x,y)\bullet los_i(x,y)-obs_i(x,y)]^2$$

u_i is a theoretical calculation of ground surface deformation vector (i=1, 2, 3)
los_i is the InSAR line-of-sight vector
obs_i is the observed deformation (InSAR image)
(x, y) is the image coordinate

Non-linear inversion!!!!

Find the best-fitting Model Parameters
Grid Search: A Simple Approach

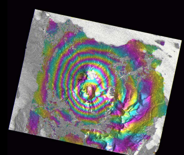
1. Loop through model parameters
2. calculate the residual (observed – modeled) for each set of model parameters
3. Find the set of model parameters that renders the smallest residual

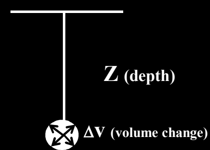
→ best-fitting model parameters

Next Week: A Jupyter Notebook Lab for Estimating Source Parameters

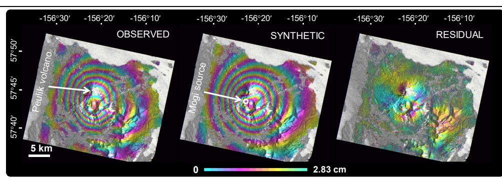
What we will do in the lab:

- We will define a search space for source model location
- We will assume that source depth and magma volume change are known and fixed
- **For each set of x and y coordinate parameters:**
  - We will run a forward model to produce *predicted surface deformation* results
  - Calculate difference (residuals) between predicted and measured deformation
- **Best fitting model parameters are those that minimize residuals between observations & model prediction**





Mt. Peulik Example



- Spherical Point Source Model (Mogi Source)

$$u_i(x_1 - \mathbf{x}'_1, x_2 - \mathbf{x}'_2, -\mathbf{x}'_3) = \mathbf{C}\frac{x_i - \mathbf{x}'_i}{|R^3|}$$

Where $\mathbf{x}'_i$ is source location, $\mathbf{C}$ is a combination of material properties and source strength, and R is the distance from the source to the surface location

- Best fit Source parameters:
  - Depth: $6.5 \pm 0.2km$; Volume change: $0.043 \pm 0.002km^3$

What's Next?

- **This is what awaits next:**
  - **Next**: Lab on Mogi source inversion from InSAR