InSAR Helps us Measure Surface Displacement

Example: Mt. Peulik volcano, Alaska

- InSAR gives us a geodetic measurement in the sense that it provides information about a pixel's location in the world and about its movement.
- InSAR does not tell us *why* the surface is moving in the observed way.
- To determine the geophysical cause of the observed motion, we need to combine our InSAR observables with geophysical models.
The Deformation Modeling Problem

deformation: what we see (InSAR)
magma dynamics: what we want to know

Estimate source characteristics from InSAR deformation data

\( \mathbf{G} \cdot \mathbf{x} = \mathbf{d} \)

- If the covariance matrix for errors in the observation (\( \mathbf{d} \)) is \( \Sigma_d \), then the weighted least-squares (maximum likelihood) solution for \( \mathbf{x} \) is

  \[ \mathbf{x} = (\mathbf{G}^T \cdot \Sigma_d^{-1} \cdot \mathbf{G})^{-1} \cdot \mathbf{G}^T \cdot \Sigma_d^{-1} \cdot \mathbf{d} \]

  and the covariance matrix for the estimated vector components is

  \[ \Sigma_x = \sigma^2 (\mathbf{G}^T \cdot \Sigma_d^{-1} \cdot \mathbf{G})^{-1} \]

- In the case where we assume that observation errors are independent and have equal standard deviations, \( \sigma \), we get

  \[ \Sigma_x = \sigma^2 (\mathbf{G}^T \cdot \mathbf{G})^{-1} \]

  - The square roots of the diagonal terms of \( \Sigma_x \) are the standard errors of the estimated parameters

Solving for Model Parameters using Model Inversion
What is the Forward Model in Volcano Deformation?

Predicts deformation $\{u\}$ caused by magma intrusion
(relates magma intrusion to deformation)

\[ \mu \nabla^2 u + \frac{\partial^2 u}{\partial x \partial y} = -F \]

What Is the Forward Model?

Simple Model: Inflating Point Source Model

- A component of deformation vector $\{u\}$ and the displacement at the free surface $z = 0$ takes the form
  \[ u_i(x_i - x_s) = C (x_i - x_s) - R_i \]
  - $x_s$ is a source location, $C$ is a combination of material properties and source strength, and $R$ is the distance from the source to the surface location.

- $C$ is defined as follows:
  \[ C = \Delta P \left( 1 - \frac{\nu}{\eta} \right) - \Delta V \frac{(1-\nu)}{2} \]
  - $\Delta P$ - change in pressure of magma chamber
  - $\Delta V$ - change in volume of magma chamber
  - $\nu$ - Poisson's ratio (material property)
  - $\eta$ - radius of the sphere
  - $G$ - shear modulus of country rock (material property)

Think – Pair – Share:

- Limitations of Mogi Models
  - Let’s look at the Mogi model equations more time
    \[ u_i(x_i - x_s) = C \frac{\Delta P}{\eta} \]
    - Activity 1: Discuss the limitations that may be brought on by how the variables $\nu$ and $G$ are used in these equations.

- Activity 2: Discuss the limitations that may be brought on by how the source geometry is captured in the equations.
Forward Model: Inflating Point Source

\[ \alpha \ll d \]

D. Dzurisin, 2007

Courtesy of M. Lisowski

\[ \alpha/d = 0.4 \]
\[ \alpha/d = 0 \text{ (point source)} \]

Forward Model: Sill Model
Forward Model: Dike Model

Ultimate Goal of Deformation Modeling:

\[
\text{Minimize } \sum [u_i(x, y) \cdot \text{los}_i(x, y) - \text{obs}_i(x, y)]^2
\]

- \(u_i\) is a theoretical calculation of ground surface deformation vector \((i=1, 2, 3)\)
- \(\text{los}_i\) is the InSAR line-of-sight vector
- \(\text{obs}_i\) is the observed deformation (InSAR image)
- \((x, y)\) is the image coordinate

Non-linear inversion!!!!

Find the best-fitting Model Parameters

1. Loop through model parameters
2. Calculate the residual (observed – modeled) for each set of model parameters
3. Find the set of model parameters that renders the smallest residual

\(\Rightarrow\) best-fitting model parameters
Next Week: A Jupyter Notebook Lab for Estimating Source Parameters

What we will do in the lab:
- We will define a search space for source model location
- We will assume that source depth and magma volume change are known and fixed
- For each set of x and y coordinate parameters:
  - We will test forward model to produce predicted surface deformation results
  - Calculate difference (residuals) between predicted and measured deformation
- Best fitting model parameters are those that minimize residuals between observations & model prediction

Mt. Peulik Example

- Spherical Point Source Model (Mogi Source)

\[ u_i(x_1 - x_i, x_2 - y_i) = u \left( \frac{x - x_i}{R^2} \right) \]

Where \( x_i \) is source location, \( u \) is a combination of material properties and source strength, and \( R \) is the distance from the source to the surface location.

Best fit Source parameters:
- Depth: 6.1 ± 0.2 km; Volume change: 0.043 ± 0.002 km³

What's Next?

- This is what awaits next:
  - Next: Lab on Mogi source inversion from InSAR